Appendix A: Spectrum of Annual Extreme Flows

At a given site, the time stream of daily flows may be partitioned into annual sets consisting of $\eta(t)$ events. An annual sequence is defined as begining with the daily flow for October 1 in year $t-1$ and ending with the daily flow for September 30 in year t, the yearly period being the "water year".

Let $\left\{x_{1,t}, \ldots, x_{\eta(t),t}\right\}$ denote the annual sequence of $\eta(t)$ daily flows in year t at an arbitrary site. The $k-day$ moving average is defined by

$$x_{t,j} = \sum_{i=j}^{k-1+j} x_{t,i} / k \tag{A-1}$$

where $j = 1, 2, \ldots, \eta(t) - k$ and $k = 1, 2, \ldots, \eta(t)$. The annual $k-day$ low, $x_{(k)}$, and high, $x^{(k)}$, flows are given by

$$x_{t,(k)} = \min_j \left\{x_{t,j}(k)\right\} \tag{A-2}$$

$$x_t^{(k)} = \max_j \left\{x_{t,j}(k)\right\} \tag{A-3}$$

Let $x_t^{(k=0)}$ denote the instantaneous peak flow. The instantaneous "trough" discharge, $x_{t,(k=0)}$, is ill defined and therefore it is not considered. A stream may experience not just an instantaneous dryness, but sustained dryness over a period of several days. Sustained dryness may be experienced in one or more years over the observational period of n years.

For $k = \eta(t)$,

$$x_{(\eta(t))} = x^{(\eta(t))} \tag{A-4}$$

i.e. the annual $\eta(t) - day$ low and high flows are equal, both being the annual mean flow.

Let K denote the set of the values of k:

$$K = \left\{ \underset{\text{Low Flow}}{1, 2, \ldots, \eta(t) - 1}, \overset{\text{Mean Flow}}{\underset{\text{Mean Flow}}{\eta(t)}}, \overset{\text{High Flow}}{\eta(t) - 1, \ldots, 1, 0} \right\} \tag{A-5}$$

The number of elements belonging to K is denoted as $\nu(K)$. If all integer values of k, where $1 \leq k \leq \eta(t)$ and $k = 0$ are considered, then $\nu(K) = 730$. The set K may be written as

$$K = K_L \cup K_M^M \cup K^H \tag{A-6}$$

where

$$K_L = \left\{1, 2, \ldots, \underset{\textit{Low Flow}}{\eta(t)} - 1\right\} \tag{A-7}$$

$$K_M^M = \left\{\underset{\textit{Mean Flow}}{\overset{\textit{Mean Flow}}{\eta(t)}}\right\} \tag{A-8}$$

$$K^H = \left\{\overset{\textit{High Flow}}{\eta(t) - 1}, \ldots, 1, 0\right\} \tag{A-9}$$

The numbers of elements in the subsets K_L, K_M^M and K^H are $\nu(K_L) = 364$, $\nu(K_M^M) = 1$ and $\nu(K^H) = 365$:

$$\begin{aligned} \nu(K) &= \nu(K_L) + \nu(K_M^M) + \nu(K^H) \\ &= 730 \end{aligned} \tag{A-10}$$

The flow spectrum is given by the $(\nu(\mathrm{K}) \text{ x } n)$ matrix

$$X = \begin{bmatrix} x_{1,(1)} & \cdots & x_{1,(\eta(1)-1)} & x_{1,(\eta(1))} = x_1^{(\eta(1))} & x_1^{(\eta(1)-1)} & \cdots & x_1^{(1)} & x_1^{(0)} \\ x_{2,(1)} & \cdots & x_{2,(\eta(2)-1)} & x_{2,(\eta(2))} = x_2^{(\eta(2))} & x_2^{(\eta(2)-1)} & \cdots & x_2^{(1)} & x_2^{(0)} \\ \vdots & \cdots & \vdots & \vdots & \vdots & \cdots & \vdots & \vdots \\ x_{n,(1)} & \cdots & x_{n,(\eta(1)-1)} & x_{n,(\eta(n))} = x_n^{(\eta(n))} & x_n^{(\eta(n)-1)} & \cdots & x_n^{(1)} & x_n^{(0)} \end{bmatrix} \tag{A-11}$$

The matrix X may be partitioned as

$$X = \begin{bmatrix} X_L & X_M^M & X^H \end{bmatrix} \tag{A-12}$$

where the partitions X_L, X_M^M and X^L are the low "end", the central "portion" and the high "end" of the flow spectrum:

$$X_L = \begin{bmatrix} x_{1(1)} & \cdots & x_{1,(\eta(1)-1)} \\ \vdots & \cdots & \vdots \\ x_{2(1)} & \cdots & x_{2,(\eta(2)-1)} \\ x_{n,(1)} & \cdots & x_{n,(\eta(n)-1)} \end{bmatrix} \qquad \text{(A-14)}$$

$$X_M^M = \begin{bmatrix} x_{1,(\eta(1))} = x_1^{(\eta(1))} \\ x_{2,(\eta(2))} = x_2^{(\eta(2))} \\ \\ x_{n,(\eta(n))} = x_n^{(\eta(n))} \end{bmatrix} \qquad \text{(A-15)}$$

$$X^H = \begin{bmatrix} x_1^{(\eta(1)-1)} & \cdots & x_1^{(1)} & x_1^{(0)} \\ x_2^{(\eta(2)-1)} & \cdots & x_2^{(1)} & x_2^{(0)} \\ \vdots & \cdots & \vdots & \vdots \\ x_n^{(\eta(n)-1)} & \cdots & x_n^{(1)} & x_n^{(0)} \end{bmatrix} \qquad \text{(A-16)}$$

The flow spectrum has no particular hydrologic significance in and of itself. It is a spectrum of extreme flows. For a given stream, the spectrum may not be fully defined. In arid regions, the low flow end of the spectrum may not be defined for values of $k \leq k^*$, where $k^* < \eta(t)$. In extremely arid regions, the spectrum loses all meaning. Where the spectrum is meaningful, it provides an ordered approach to assessing consistency in the characterization of hydrology extremes.

Each column vector of the flow spectrum X is a sequence of extreme flows. If the theory of extremes (see e.g. Galambos: 1978) held for each vector, then there would be continuity in the distribution of the flows over the values of k. The implied continuity of the theory derives from the fact that the theory yields three distinct distribution of maximum flows and three distributions of minimum flows. Whether extreme value theory holds for the vectors is arguable.

In the following discussions, the spectrum is defined for the respective subsets of K_L, K_M^M and K^H, namely

$$\tilde{K}_L = \{1, 3, 7, 14, 30, 60, 90, 180\}$$

$$\tilde{K}_M^M = K_M^M = \{\eta(t)\}$$

$$\tilde{K}^H = \{180, 90, 60, 30, 14, 7, 3, 1, 0\}$$

Appendix B: Regionalization Scheme

For an arbitrary spectral element, let $x_{t,j}$ denote the flow for year $t = 1,\ldots,n$ at site $j = 1,\ldots,M$. The flows are arrayed as

$$X = \begin{bmatrix} x_{1,1} & x_{1,2} & \cdots & x_{1,M} \\ x_{2,1} & x_{22} & \cdots & x_{2,M} \\ \vdots & \vdots & & \vdots \\ x_{n,1} & x_{n,2} & \cdots & x_{n,M} \end{bmatrix} \tag{B-1}$$

For each site, the flows ordered in magnitude from smallest

$$y_{1,j} = \min_t \left\{ x_{t,j} \right\} \tag{B-2}$$

to largest

$$y_{n,j} = \max_t \left\{ x_{t,j} \right\} \tag{B-3}$$

are arrayed as

$$Y = \begin{bmatrix} y_{1,1} & y_{1,2} & \cdots & y_{1,M} \\ y_{21} & y_{2,2} & \cdots & y_{2,M} \\ \vdots & \vdots & & \vdots \\ y_{n,1} & y_{n,2} & \cdots & y_{n,M} \end{bmatrix} \tag{B-4}$$

Thr probability distribution of the flows at site j is denoted as

$$D_j \Rightarrow \left\{ \left(F_t, y_{t,j} \right) \; t = 1,\ldots,n \right\} \tag{B-5}$$

where F_t is an assigned value of the exceedence probability of $y_{t,j}$.

Let $\tilde{y}_j$ denote the median of the flow at site j. Define $z_{t,j} = y_{t,j} / \tilde{y}_j$. For the array of normalized flows

$$Z = \begin{bmatrix} z_{1,1} & z_{1,2} & \cdots & z_{1,M} \\ z_{2,1} & z_{2,2} & \cdots & z_{2,M} \\ \vdots & \vdots & & \vdots \\ z_{n,1} & z_{n,2} & \cdots & z_{n,M} \end{bmatrix} \tag{B-6}$$

the median of each column is equal to 1. Let w_t denote the median of the $t-th$ row.

The vector

$$W = \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{B-7}$$

is the regional ordered set of normalized flows. The distribution of W is given by

$$\Delta(W) \Rightarrow \{(F_t, w_t)\colon t = 1, \dots, n\} \tag{B-8}$$

For site j, the regionalized ordered set of flow is given by

$$X_j' = \tilde{y}_j \begin{bmatrix} w_1 \\ w_2 \\ \vdots \\ w_n \end{bmatrix} \tag{B-9}$$

and the regionalized probability distribution of flow at site j is defined as

$$\begin{aligned} D(X_j') &\equiv D_j' \\ &\Rightarrow \{(F_t, \tilde{y}_j w_t)\ t = 1, \dots, n\} \end{aligned} \tag{B-10}$$

The location and scale parameters of $\Delta(W)$ and $D'(j)$ differ by a factor equal to $\tilde{y}_j$, but the skewness and the kurtosis of the distributions are the same.

The regional distributions may also be obtained indirectly through log space. Let

$$\xi_{t,j} = Ln\left(x_{t,j}\right) \tag{B-11}$$

The natural logs of the flows are arrayed as

$$\Xi = \begin{bmatrix} \xi_{1,1} & \xi_{1,2} & \cdots & \xi_{1,M} \\ \xi_{2,1} & \xi_{2,2} & \cdots & \xi_{2M} \\ \vdots & \vdots & & \vdots \\ \xi_{n,1} & \xi_{n,2} & \cdots & \xi_{n,M} \end{bmatrix} \tag{B-12}$$

For each site, the logs of the flows ordered in magnitude from smallest to largest are arrayed as

$$\Psi = \begin{bmatrix} \psi_{1,1} & \psi_{1,2} & \cdots & \psi_{1,M} \\ \psi_{2,1} & \psi_{2,2} & \cdots & \psi_{2,M} \\ \vdots & \vdots & & \vdots \\ \psi_{n,1} & \psi_{n,2} & \cdots & \psi_{n,M} \end{bmatrix} \tag{B-13}$$

Let $\tilde{\psi}_j$ denote the median of the logs of the flows at the $j-th$ site. Define $\phi_{t,j} = \psi_{t,j} \big/ \tilde{\psi}_j$. For the array

$$\Phi = \begin{bmatrix} \phi_{1,1} & \phi_{1,2} & \cdots & \phi_{1,M} \\ \phi_{2,1} & \phi_{2,2} & \cdots & \phi_{2,M} \\ \vdots & \vdots & & \vdots \\ \phi_{n,1} & \phi_{n,2} & \cdots & \phi_{n,M} \end{bmatrix} \tag{B-14}$$

each column has median equal to 1. Let ω_t denote the median of the $t-th$ row. The vector

$$\Omega = \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} \qquad \text{(B-15}$$

is the regional ordered set of normalized logs of the flows.
The distribution of Ω is defined by

$$\Delta(\Omega) \Rightarrow \left\{ (F_t, \omega_t)\!: \; t = 1,\ldots,n \right\} \tag{B-16}$$

For the $j-th$ site, the regionalized ordered set of the logs of the flows is given by

$$\Theta_j = \tilde{\psi}_j \begin{bmatrix} \omega_1 \\ \omega_2 \\ \vdots \\ \omega_n \end{bmatrix} \tag{B-17}$$

The distribution of Θ_j is defined by

$$\Delta\left(\Theta_j\right) \Rightarrow \left\{ \left(F_t, \psi_j \omega_t\right) \; t = 1,\ldots,n \right\} \tag{B-18}$$

For the $j-th$ site, the ordered set of regionalized flows in real space is given by

$$X_j'' = \begin{bmatrix} exp\left(\tilde{\psi}_j \omega_1\right) \\ exp\left(\tilde{\psi}_j \omega_2\right) \\ \vdots \\ exp\left(\tilde{\psi}_j \omega_n\right) \end{bmatrix} \tag{B-19}$$

The distribution of X_j'' is defined by

$$\begin{aligned} D\left(X_j''\right) &\equiv D_j'' \\ &\Rightarrow \left\{ \left(F_t, exp\left(\tilde{\psi}_j \omega_t\right)\right) \; t = 1,\ldots,n \right\} \end{aligned} \tag{B-20}$$

Through the process of regionalization outlined above, the flows at $j-th$ site may be represented by one of three distributions, D_j, the distribution based only on the observations at the site, (Eq. (B-5)); D_j', the distribution determined directly through regionalization in real space, (Eq. (B-10)); and D_j'', the distribution determined indirectly through regionalization in log space, (Eq. (B-20)).

The regional distributions for the spectral elements are shown in Figures B-1 through B-8 for the Upper Mississippi basin, and in Figures B-9 through B-16 for the Missouri basin. The regional distributions for elements of the low (high) side of the spectrum are shown relative to the regional distributions for the annual 1-day low (high) flows and the annual mean flows. Following hydrologic convention, the the distributions over the low end and over the high end of the spectrum are defined as

$$Prob\left[x_\kappa < X_\kappa\right] = F\left(x_\kappa\right) \tag{B-21}$$

$$Prob\left[x^\kappa > X^\kappa\right] = F\left(x^\kappa\right) \tag{B-22}$$

The high end of the spectrum relates to floods, whereby it is the right tails of the distributions that matter. The low end of the spectrum relates to conditions of dryness, whereby, it is the left tails of the distributions that matter.

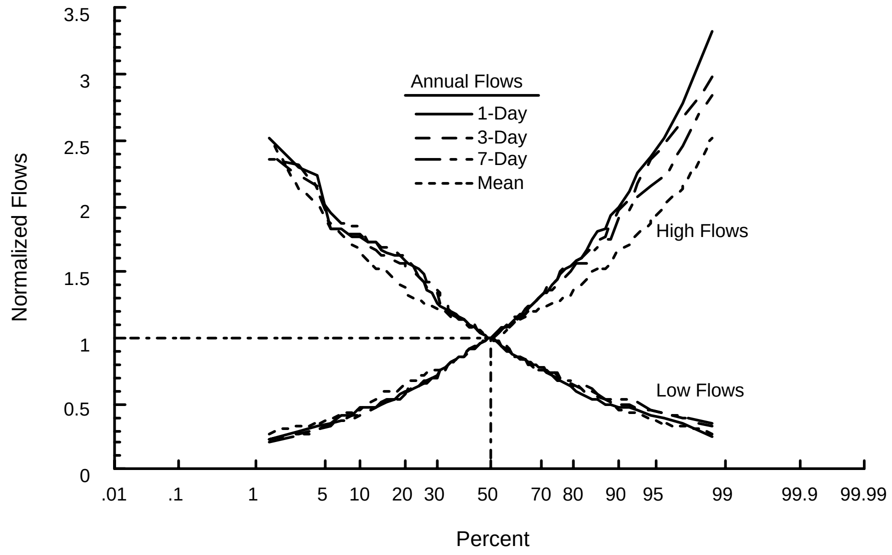


Figure B-1: Upper Mississippi Basin - Regionalized Distributions of Annual 1-Day, 3-Day and 7-Day Low and High Flows and Annual Mean Flows

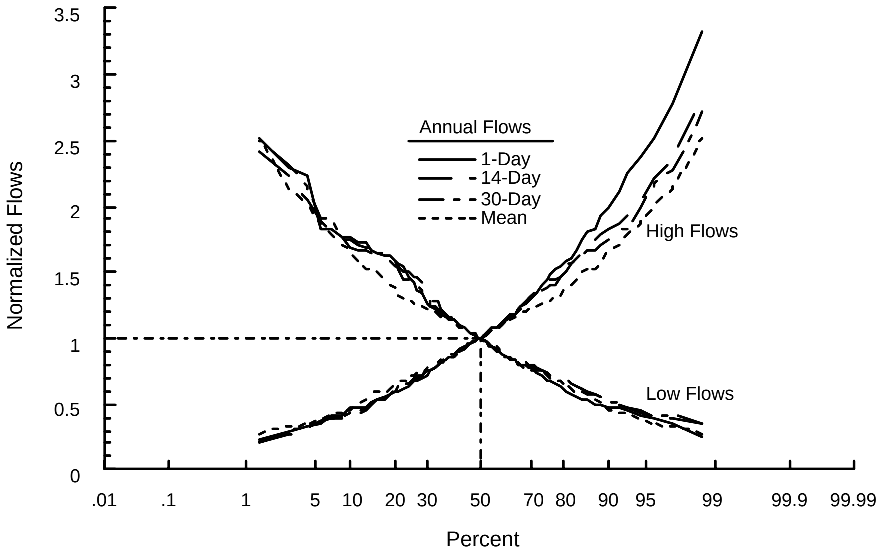


Figure B-2: Upper Mississippi Basin - Regionalized Distributions of Annual 1-Day, 14-Day and 30-Day Low andHigh Flows and Annual Mean Flows

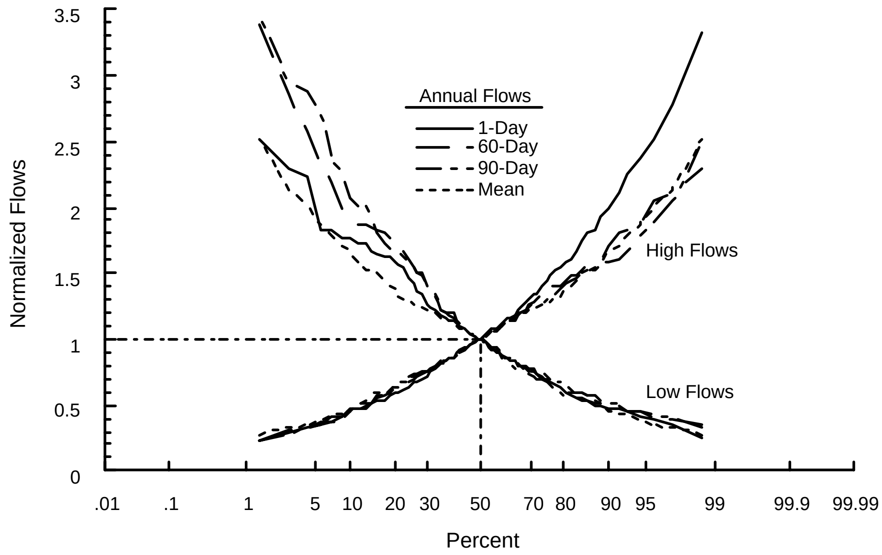


Figure B-3: Upper Mississippi Basin - Regionalized Distributions of Annual 1-Day, 60-Day and 90-Day Low and High Flows and Annual Mean Flows

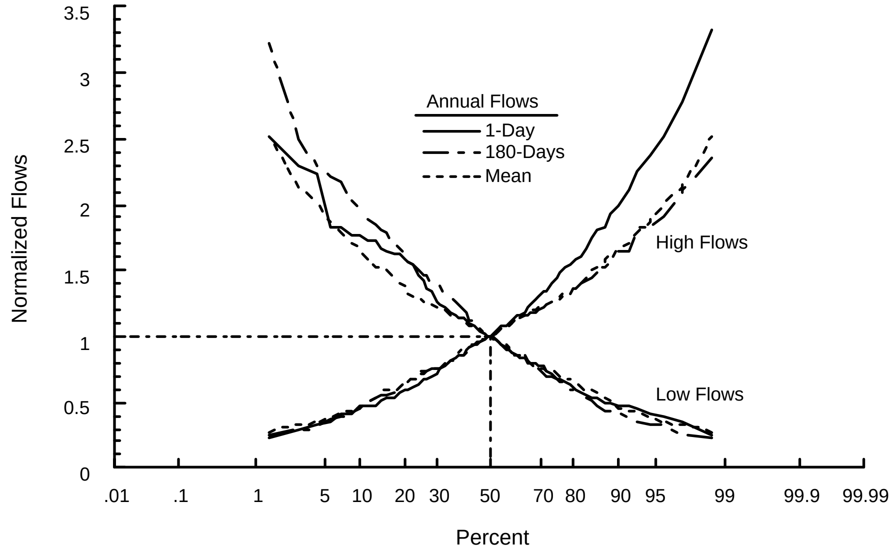


Figure B-4: Upper Mississippi Basin - Regionalized Distributions of Annual 1-Day and 180-Day Low and High Flows and Annual Mean Flows

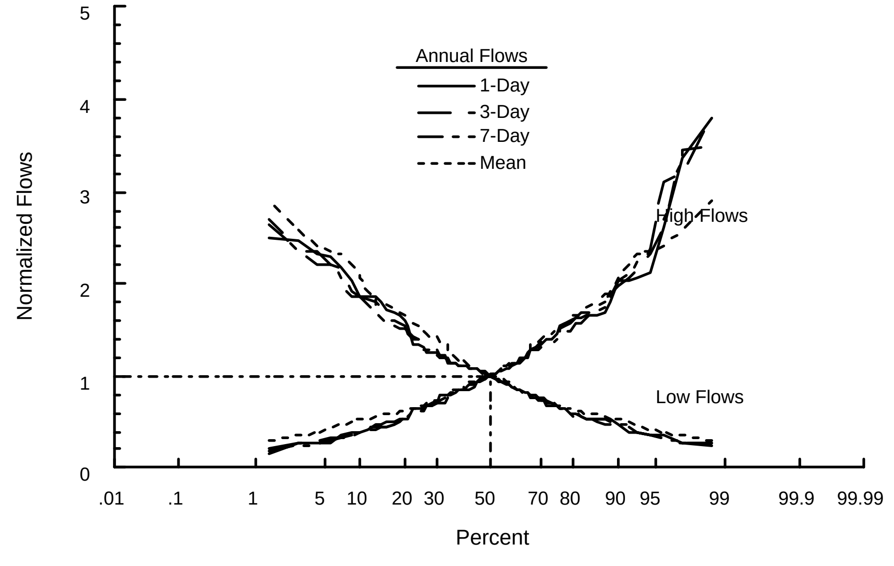


Figure B-5: Missouri Basin - Regionalized Distributions of Annual 1-Day, 3-Day and 7-Day Low and High Flows and Annual Mean Flows

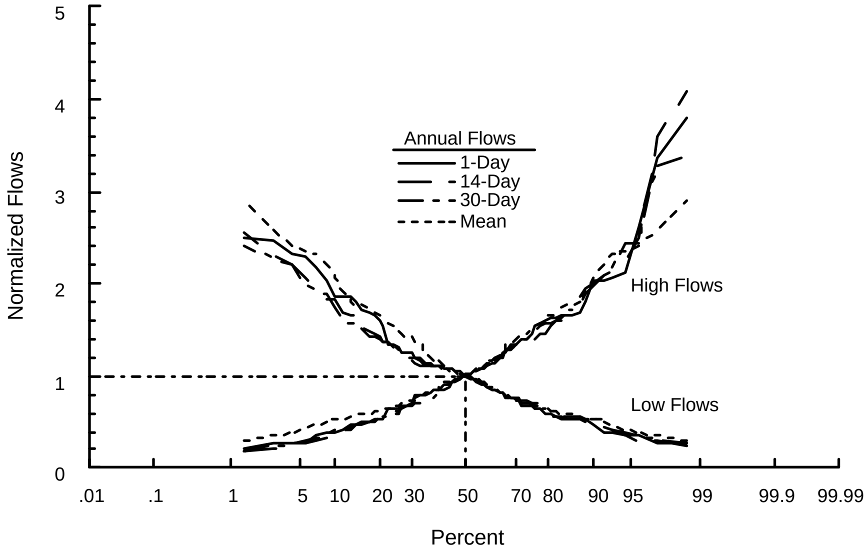


Figure B-6: Missouri Basin - Regionalized Distributions of Annual 1-Day, 14-Day and 30-Day Low and High Flows and Annual Mean Flows

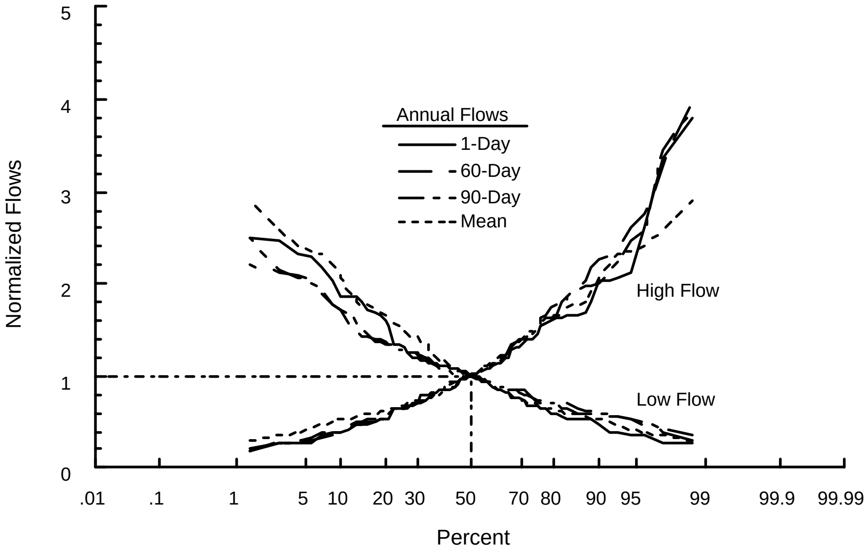


Figure B-7: Missouri Basin - Regionalized Distributions of Annual 1-Day, 60-Day and 90-Day Low and High Flows and Annual Mean Flows

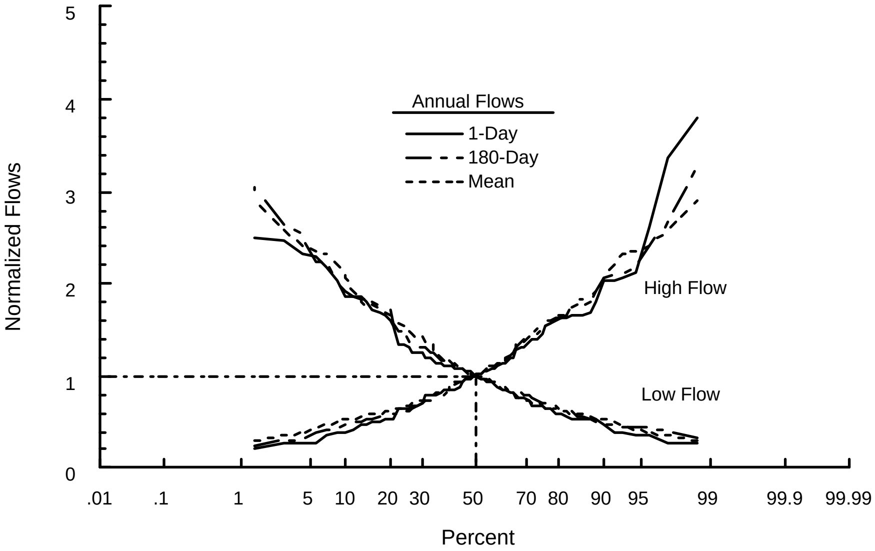


Figure B-8: Missouri Basin - Regionalized Distributions of Annual 1-Day and 180-Day Low and High Flows and Annual Mean Flows

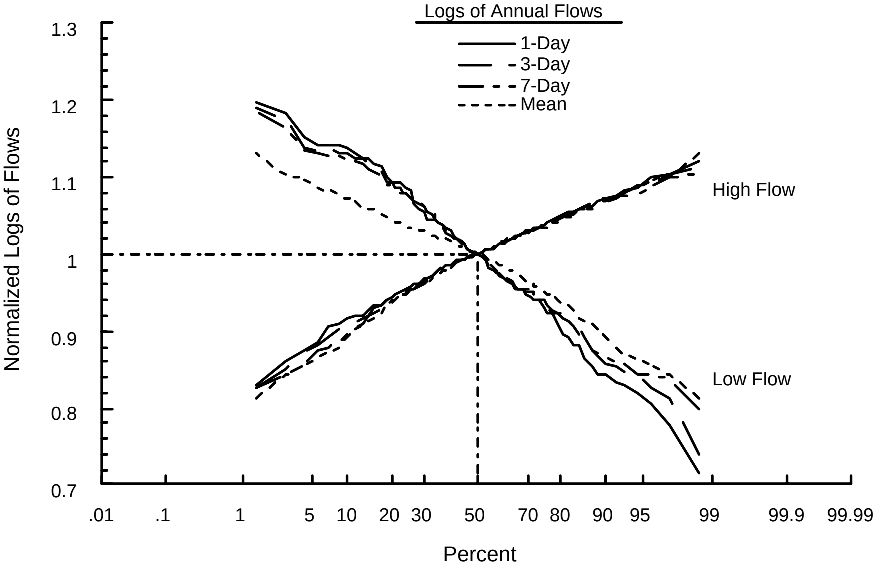


Figure B-9: Upper Mississippi Basin - Regionalized Distributions of Logs of 1-Day, 3-Day and 7-Day Low and High Flows and Logs of Annual Mean Flows

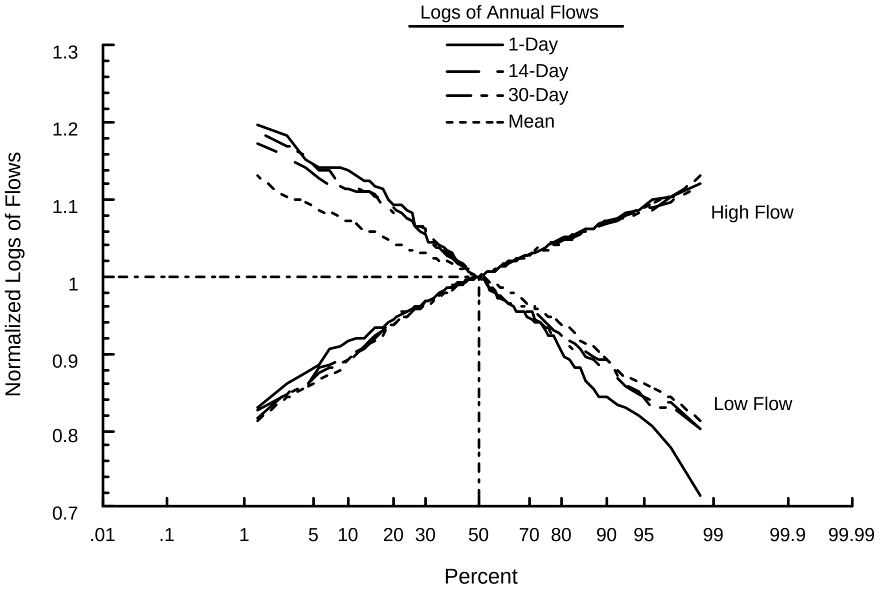


Figure B-10: Upper Mississippi Basin - Regionalized Distributions of Logs of Annual 1-Day, 14-Day and 30-Day Low and High Flows and Logs of Annual Mean Flows

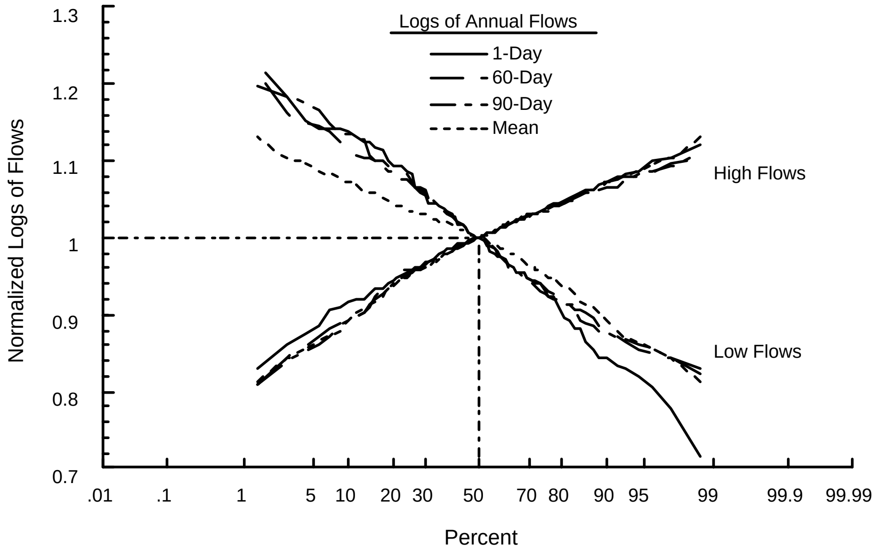


Figure B-11: Upper Mississippi Basin - Regionalized Distributions of Logs of Annual 1-Day, 60-Day and 90-Day Low and High Flows and Logs of Annual Mean Flows

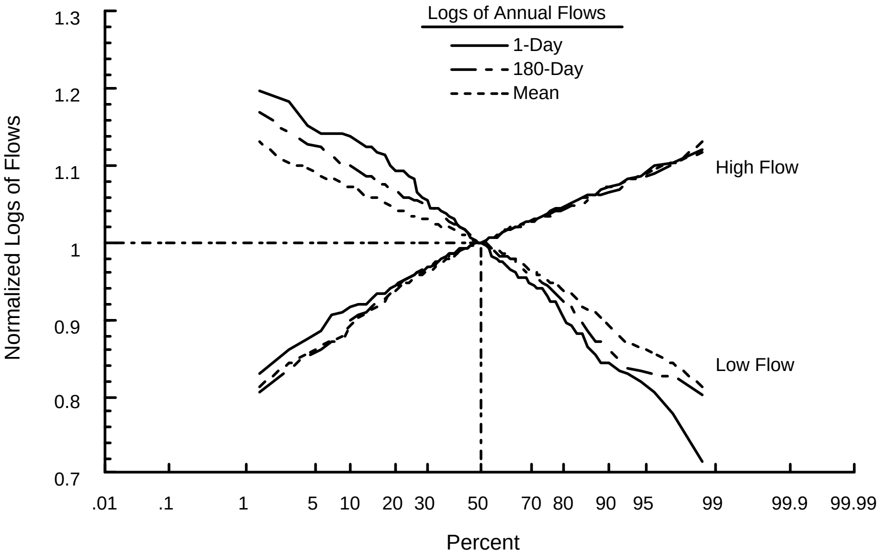


Figure B-12: Upper Mississippi Basin - Regionalized Distributions of Logs of Annual 1-Day and 180-Day Low and High Flows and Logs of Annual Mean Flows

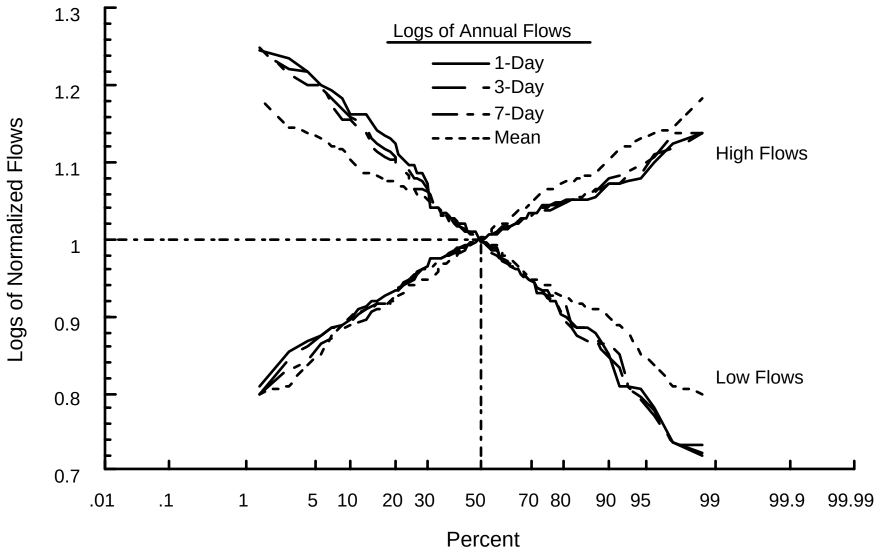


Figure B-13: Missouri Basin - Regionalized Distributions of Logs of Annual 1-Day, 3-Day and 7-Day Low and High Flows and Logs of Annual Mean Flows

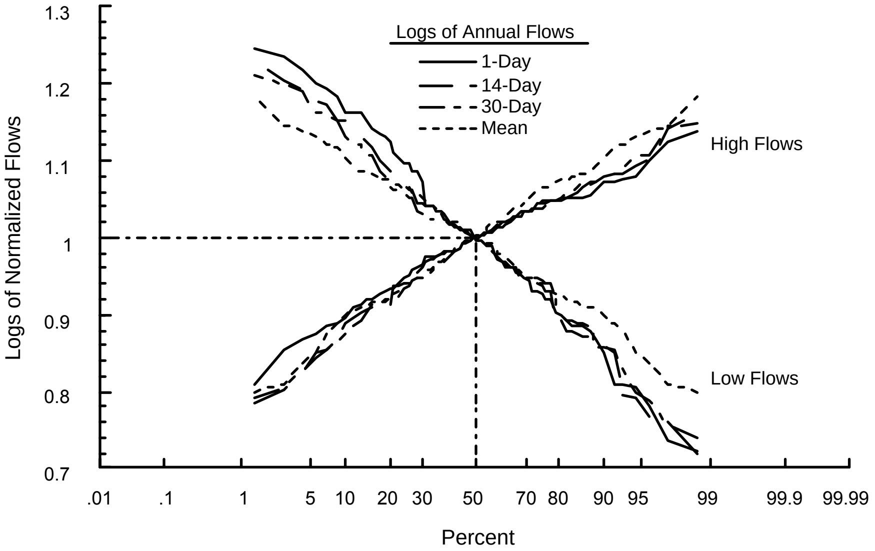


Figure B-14: Missouri Basin - Regionalized Distributions of Logs of Annual 1-Day, 14-Day and 30-Day Low and High Flows and Logs of Annual Mean Flows

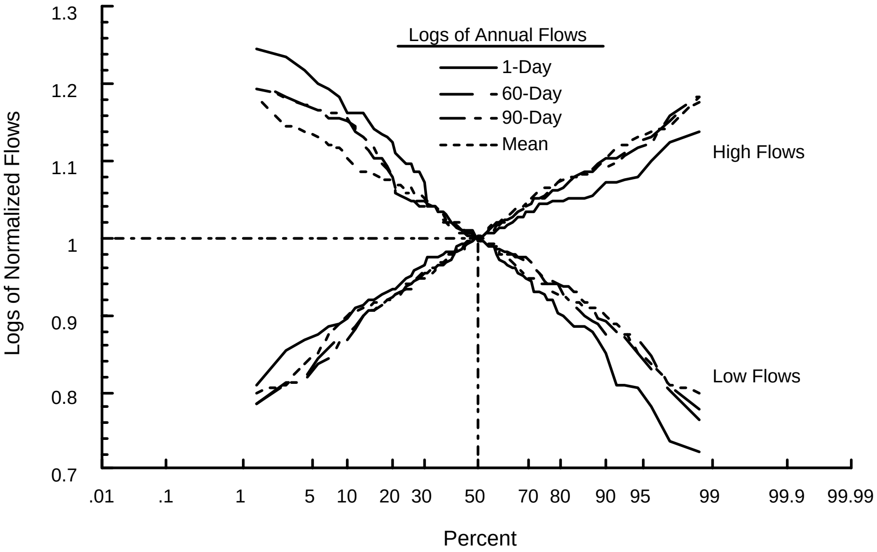


Figure B-15: Missouri Basin - Regionalized Distributions of Logs of Annual 1-Day, 60-Day and 90-Day Low and High Flows and Logs of Annual Mean Flows

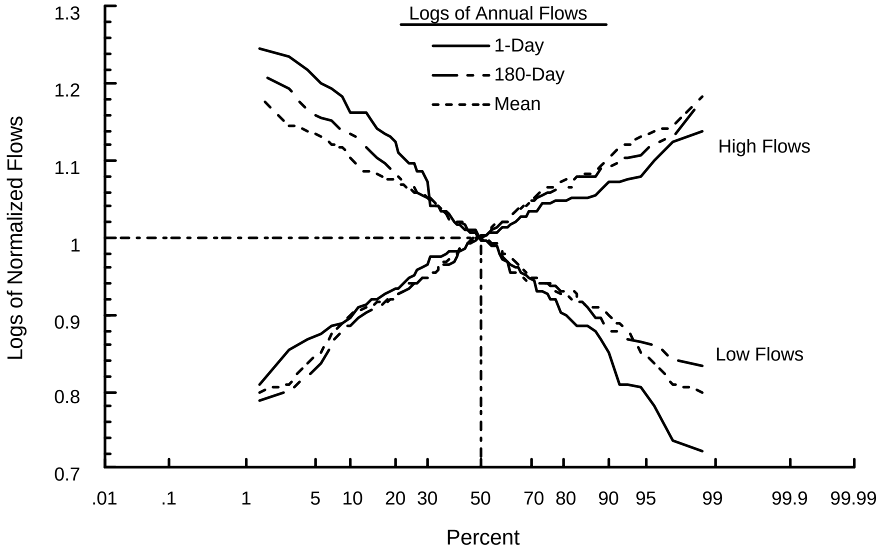


Figure B-16: Missouri Basin - Regionalized Distributions of Logs of Annual 1-Day and 180-Day Low and High Flows and Logs of Annual Mean Flow

Appendix C: Statistical Description

The at-site sequences and the regionalized sequences in both real space and log space are described in terms of the dimensionless parameters, the coefficients of variation, Cv, the coefficients of skewness, Sk, and the coefficients of kurtosis, Ku. relative to the elements of the flow spectrum. The spectrum is partitioned into 17 elements, whereby, there are a total of 544 at-site sequences, of which 357 relate to the Upper Mississippi basin and 187, to the Lower Missouri basin.

Description of At-Site Sequences

Table C-1a: Spectral Coefficient of Variation (CV) in Real Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	0.494	0.301	0.263	0.263	0.269	0.269	0.284	0.383	0.308	0.329	0.337	0.346	0.382	0.406	0.399	0.389	0.386
Jump	576	0.424	0.394	0.399	0.419	0.515	0.530	0.524	0.596	0.331	0.317	0.355	0.352	0.388	0.438	0.504	0.574	0.615
Black	749	0.652	0.652	0.638	0.634	0.637	0.645	0.681	0.585	0.388	0.357	0.367	0.347	0.376	0.417	0.460	0.484	0.542
Maquaketa	1553	0.444	0.432	0.437	0.440	0.428	0.437	0.459	0.453	0.449	0.481	0.483	0.513	0.530	0.550	0.556	0.577	0.601
Mississippi	85600	0.270	0.277	0.296	0.299	0.304	0.311	0.333	0.344	0.298	0.312	0.321	0.331	0.350	0.363	0.363	0.358	0.354
Rock	3340	0.748	0.630	0.569	0.548	0.547	0.568	0.574	0.470	0.410	0.405	0.420	0.441	0.446	0.438	0.428	0.418	0.409
Sugar	923	0.393	0.347	0.328	0.315	0.297	0.318	0.328	0.328	0.309	0.342	0.368	0.401	0.466	0.525	0.527	0.590	0.655
Pecatonica	1326	0.500	0.456	0.443	0.438	0.423	0.434	0.444	0.427	0.400	0.431	0.419	0.450	0.488	0.517	0.519	0.554	0.577
Cedar	6510	0.557	0.547	0.544	0.555	0.559	0.600	0.710	0.617	0.606	0.651	0.619	0.602	0.585	0.598	0.595	0.624	0.628
Skunk	4303	0.915	0.918	0.914	0.897	0.890	1.051	0.963	0.711	0.628	0.609	0.604	0.601	0.571	0.538	0.549	0.558	0.541
Mississippi	119000	0.380	0.379	0.369	0.360	0.362	0.374	0.387	0.382	0.352	0.369	0.371	0.374	0.379	0.360	0.353	0.350	0.348
Des Moines	5452	0.944	0.927	0.925	0.944	0.973	0.974	1.140	0.881	0.752	0.778	0.731	0.720	0.688	0.669	0.657	0.653	0.645
Raccoon	3441	1.022	1.044	1.035	1.031	1.124	1.028	1.151	0.866	0.703	0.708	0.691	0.672	0.653	0.646	0.631	0.650	0.664
Iroquois	2091	0.735	0.706	0.672	0.691	0.763	1.607	1.030	0.577	0.436	0.398	0.393	0.421	0.422	0.419	0.428	0.430	0.432
Kankakee	2294	0.274	0.269	0.268	0.271	0.279	0.376	0.394	0.346	0.296	0.293	0.292	0.303	0.294	0.288	0.298	0.321	0.336
Spoon	1636	1.168	1.137	1.090	1.038	0.914	1.313	1.156	0.744	0.531	0.492	0.463	0.476	0.485	0.491	0.500	0.482	0.498
La Moines	1293	0.827	0.821	0.841	0.854	0.908	1.115	1.397	0.766	0.566	0.533	0.544	0.556	0.598	0.598	0.565	0.564	0.566
Meramec	781	0.265	0.263	0.265	0.267	0.279	0.379	0.422	0.547	0.465	0.492	0.493	0.494	0.535	0.590	0.655	0.718	0.701
Bourbeuse	808	0.364	0.363	0.359	0.353	0.368	0.883	0.970	0.666	0.529	0.502	0.504	0.487	0.500	0.530	0.578	0.602	0.626
Big	917	0.399	0.385	0.371	0.361	0.391	0.505	0.606	0.542	0.460	0.471	0.455	0.468	0.481	0.515	0.586	0.625	0.616
Meramec	3788	0.322	0.318	0.316	0.315	0.350	0.464	0.589	0.552	0.463	0.469	0.467	0.462	0.485	0.514	0.572	0.597	0.620
Mean		0.576	0.551	0.540	0.538	0.551	0.675	0.693	0.561	0.461	0.464	0.462	0.468	0.481	0.496	0.511	0.529	0.541
Standard Deviation		0.271	0.275	0.271	0.267	0.270	0.373	0.335	0.167	0.133	0.134	0.120	0.113	0.102	0.099	0.101	0.113	0.117
Yellowstone	2623	0.233	0.233	0.235	0.230	0.229	0.229	0.219	0.196	0.216	0.246	0.263	0.274	0.292	0.307	0.301	0.282	0.270
Clarks Fork	1154	0.342	0.336	0.318	0.314	0.301	0.218	0.172	0.179	0.219	0.243	0.245	0.246	0.261	0.280	0.263	0.238	0.239
Yellowstone	11795	0.300	0.284	0.252	0.226	0.209	0.191	0.180	0.173	0.240	0.281	0.298	0.306	0.319	0.330	0.330	0.311	0.301
Big Sioux	8424	1.163	1.155	1.142	1.126	1.132	1.137	1.193	0.913	0.943	0.966	0.960	0.929	0.955	1.015	1.033	1.019	1.013
North Platte	1431	0.440	0.448	0.440	0.423	0.391	0.372	0.360	0.395	0.433	0.471	0.494	0.508	0.503	0.493	0.474	0.463	0.451
Bear	164	0.486	0.459	0.423	0.410	0.383	0.349	0.329	0.367	0.524	0.623	0.712	0.752	0.779	0.819	0.840	0.879	0.900
Elkhorn	6900	0.647	0.624	0.612	0.600	0.578	0.575	0.569	0.579	0.587	0.638	0.694	0.676	0.706	0.796	0.855	0.910	0.898
Nishnabottna	2806	0.957	0.942	0.946	0.919	0.918	0.915	0.940	0.766	0.691	0.694	0.725	0.724	0.733	0.694	0.655	0.631	0.633
Grand	2250	1.018	0.999	0.985	1.035	1.056	1.328	1.249	0.906	0.740	0.722	0.767	0.791	0.862	0.740	0.713	0.651	0.572
Thompson	1670	1.004	0.981	0.961	0.964	1.038	1.439	1.182	0.807	0.653	0.628	0.670	0.670	0.725	0.661	0.668	0.683	0.640
Gasconade	2840	0.245	0.246	0.248	0.252	0.276	0.328	0.389	0.513	0.447	0.467	0.474	0.490	0.510	0.534	0.579	0.623	0.655
Mean		0.621	0.610	0.596	0.591	0.592	0.644	0.617	0.527	0.517	0.543	0.573	0.579	0.604	0.606	0.610	0.608	0.597
Standard Deviation		0.351	0.347	0.347	0.353	0.369	0.474	0.437	0.288	0.235	0.227	0.234	0.230	0.240	0.238	0.249	0.263	0.266

Table C-1b: Spectral Coefficient of Skewness (Sk) in Real Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	-0.038	0.493	0.413	0.247	0.251	0.428	0.724	0.877	0.207	0.210	0.221	0.420	0.488	0.389	0.325	0.318	0.372
Jump	576	0.414	0.420	0.472	0.655	2.084	1.733	1.208	0.779	0.181	0.352	0.458	0.526	0.959	1.455	2.054	2.968	3.613
Black	749	0.749	0.782	0.770	0.779	0.905	1.187	1.886	0.933	0.495	0.385	0.638	0.059	0.116	0.399	0.485	0.639	1.359
Maquaketa	1553	1.037	0.910	0.875	0.733	0.495	0.582	0.689	1.082	1.148	1.222	0.967	1.024	1.120	1.190	0.891	0.605	0.765
Mississippi	85600	-0.075	-0.023	0.204	0.369	0.486	0.507	0.662	0.718	0.236	0.436	0.152	0.168	0.517	0.610	0.577	0.548	0.559
Rock	3340	1.663	1.443	1.363	1.257	1.177	1.131	1.067	0.837	0.591	0.506	0.433	0.477	0.623	0.622	0.615	0.609	0.545
Sugar	923	0.389	0.376	0.417	0.478	0.441	0.580	0.562	0.914	0.660	0.686	0.436	0.469	0.789	1.124	0.948	1.193	1.282
Pecatonica	1326	0.715	0.762	0.745	0.790	0.764	0.780	0.826	1.260	1.292	1.341	0.851	0.908	0.949	1.051	0.880	1.087	1.180
Cedar	6510	1.044	0.953	0.811	0.835	0.746	0.915	1.719	1.311	1.765	2.192	1.493	1.282	0.875	0.929	0.748	0.767	0.796
Skunk	4303	1.480	1.523	1.422	1.343	1.218	2.223	1.656	0.935	1.515	1.393	1.185	1.023	0.801	0.420	0.660	0.939	1.032
Mississippi	119000	0.855	0.696	0.594	0.635	0.673	0.635	0.618	0.666	0.834	1.131	0.603	0.625	0.932	0.690	0.581	0.548	0.607
Des Moines	5452	1.580	1.575	1.615	1.790	1.808	1.516	2.618	1.812	1.856	2.000	1.549	1.491	1.179	1.163	1.124	1.244	1.286
Raccoon	3441	2.983	2.981	2.828	2.830	3.240	2.936	2.791	1.757	1.366	1.258	1.006	0.897	0.926	0.961	1.023	1.231	1.402
Iroquois	2091	2.113	2.057	1.941	2.059	1.719	6.152	3.325	0.835	0.651	0.112	0.075	0.356	0.497	0.238	0.229	0.311	0.375
Kankakee	2294	1.069	1.091	1.027	1.076	1.176	1.718	1.422	0.396	0.233	0.044	-0.052	0.162	0.137	0.124	0.215	0.528	0.633
Spoon	1636	3.968	3.933	3.739	3.313	2.572	4.847	3.583	1.734	1.321	0.693	0.399	0.467	0.527	0.419	0.552	0.637	0.851
La Moines	1293	2.142	2.026	2.320	2.363	2.496	3.491	4.002	1.115	0.603	0.122	0.285	0.471	0.807	0.887	0.538	0.733	0.717
Meramec	781	0.545	0.590	0.636	0.662	0.949	2.127	1.352	1.314	0.822	0.773	0.841	0.692	0.973	1.004	1.237	1.481	1.401
Bourbeuse	808	0.736	0.736	0.749	0.741	0.776	4.290	2.926	0.769	0.802	0.607	0.678	0.639	0.559	0.866	1.334	1.700	2.336
Big	917	0.701	0.706	0.775	0.722	0.992	1.544	1.674	0.793	0.572	0.593	0.584	0.534	0.657	0.799	1.500	1.398	1.310
Meramec	3788	0.680	0.662	0.645	0.594	1.131	1.651	1.994	0.833	0.573	0.492	0.516	0.383	0.589	0.542	0.896	1.063	1.341
Mean		1.179	1.176	1.160	1.156	1.243	1.951	1.776	1.032	0.844	0.788	0.634	0.623	0.715	0.756	0.829	0.978	1.132
Standard Deviation		0.980	0.936	0.892	0.839	0.795	1.567	1.052	0.377	0.508	0.598	0.431	0.367	0.284	0.352	0.447	0.599	0.734
Yellowstone	2623	0.339	0.397	0.476	0.400	0.350	0.373	0.330	0.214	0.464	0.427	0.473	0.596	0.697	0.920	0.912	0.883	0.745
Clarks Fork	1154	-0.046	-0.083	-0.193	-0.337	-0.440	-0.739	-0.259	0.240	0.565	0.520	0.423	0.461	0.453	0.584	0.532	0.426	0.560
Yellowstone	11795	0.332	0.363	0.424	0.243	0.033	0.009	0.157	0.188	0.379	0.401	0.388	0.472	0.574	0.790	0.835	0.733	0.662
Big Sioux	8424	1.849	1.824	1.806	1.793	1.935	2.026	2.141	1.848	2.077	2.241	2.032	1.865	1.949	2.136	2.536	2.384	2.283
North Platte	1431	1.003	1.111	1.180	1.161	1.159	0.978	0.852	1.469	0.602	0.568	0.520	0.591	0.780	0.744	0.621	0.625	0.494
Bear	164	0.132	0.218	0.353	0.410	0.378	0.393	0.498	1.064	0.934	1.020	1.204	1.280	1.286	1.424	1.500	1.785	1.811
Elkhorn	6900	1.664	1.515	1.349	1.218	1.286	1.422	1.429	1.182	1.261	1.402	1.477	1.153	1.038	1.443	1.656	2.203	2.793
Nishnabottna	2806	1.994	1.874	1.886	1.797	1.960	1.943	1.971	1.677	1.685	1.621	1.767	1.817	2.148	1.806	1.391	1.274	1.549
Grand	2250	2.351	2.183	2.136	2.463	2.250	3.343	2.807	1.340	1.705	1.769	1.905	2.219	2.971	2.139	1.941	1.467	1.386
Thompson	1670	2.986	2.867	2.640	2.342	2.513	3.570	2.779	0.913	0.944	0.886	1.248	1.343	2.213	2.082	1.930	1.691	1.317
Gasconade	2840	0.407	0.394	0.371	0.406	0.828	1.008	1.264	0.886	0.667	0.519	0.766	0.925	0.802	0.820	1.169	1.268	1.554
Mean		1.183	1.151	1.130	1.081	1.114	1.302	1.270	1.002	1.026	1.034	1.109	1.156	1.356	1.354	1.366	1.340	1.378
Standard Deviation		1.031	0.963	0.907	0.929	0.972	1.341	1.056	0.585	0.577	0.634	0.628	0.613	0.832	0.611	0.624	0.641	0.737

Table C-1c: Spectral Coefficient of Kurtosis (Ku) in Real Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	-0.511	-0.102	-0.093	-0.424	-0.517	-0.396	0.188	-0.161	-0.295	-0.345	-0.237	0.424	0.183	-0.526	-0.447	-0.338	-0.194
Jump	576	-0.344	-0.506	-0.499	-0.063	8.287	4.837	1.673	0.131	-0.580	-0.184	-0.436	-0.335	1.449	3.755	7.569	15.079	20.950
Black	749	0.201	0.214	0.196	0.148	0.495	1.398	6.865	1.290	0.047	0.244	1.547	0.089	-0.581	-0.117	-0.116	0.480	3.253
Maquaketa	1553	1.385	1.172	1.087	0.480	-0.094	-0.062	0.020	2.039	2.453	2.870	1.476	1.360	1.675	1.817	0.351	-0.846	-0.285
Mississippi	85600	-0.259	-0.254	-0.356	-0.240	-0.485	-0.671	-0.129	0.054	0.147	1.442	0.023	-0.194	0.286	0.455	0.458	0.497	0.568
Rock	3340	3.672	2.663	2.301	1.780	1.257	1.181	0.548	0.776	0.126	0.155	-0.270	-0.415	-0.216	-0.202	-0.157	-0.140	-0.225
Sugar	923	-0.891	-0.918	-0.966	-0.952	-0.958	-0.629	-0.712	0.871	0.557	0.766	-0.361	-0.511	0.375	1.505	0.829	1.798	1.503
Pecatonica	1326	-0.614	-0.503	-0.502	-0.315	-0.380	-0.439	-0.215	2.262	3.478	3.989	1.601	1.523	1.085	1.142	0.254	0.805	1.072
Cedar	6510	0.757	0.396	-0.105	-0.151	-0.464	0.059	4.022	2.504	6.044	8.974	4.161	3.044	0.969	1.025	0.319	-0.012	0.004
Skunk	4303	2.011	1.970	1.513	1.232	0.797	5.923	3.109	0.426	4.913	4.266	2.843	2.078	0.822	-0.285	0.046	1.007	1.307
Mississippi	119000	2.795	1.602	0.731	0.607	0.214	-0.232	-0.357	0.309	2.444	4.315	1.470	1.512	2.964	2.049	1.602	1.330	1.446
Des Moines	5452	1.839	1.881	2.121	3.024	2.883	1.559	9.357	4.745	5.680	6.352	3.261	3.192	1.680	1.701	1.568	2.148	2.230
Raccoon	3441	10.336	10.285	9.167	9.270	13.023	11.497	8.879	3.527	2.237	1.563	0.405	0.048	0.463	0.836	1.376	2.241	2.956
Iroquois	2091	6.342	6.175	5.669	6.828	3.177	45.053	17.582	1.967	1.499	-0.198	-0.249	0.045	0.535	-0.604	-0.913	-0.828	-0.750
Kankakee	2294	1.757	1.883	1.668	1.902	2.390	4.257	2.894	0.053	-0.068	-0.197	-0.392	-0.084	0.254	-0.035	0.003	0.803	1.083
Spoon	1636	19.944	19.772	17.986	14.173	8.775	30.784	18.561	3.894	3.588	0.625	-0.233	-0.253	0.084	-0.218	0.053	0.335	0.577
La Moines	1293	5.489	4.822	7.174	7.974	8.191	18.126	20.858	1.254	0.474	-0.984	-0.699	0.000	0.885	1.199	-0.277	-0.061	-0.153
Meramec	781	0.092	0.185	0.236	0.411	1.451	7.417	1.440	2.426	0.578	0.292	0.928	0.101	0.797	0.583	1.321	2.395	2.123
Bourbeuse	808	0.257	0.251	0.333	0.279	0.265	24.443	10.708	-0.305	0.647	0.281	0.780	0.666	-0.153	0.988	2.630	4.955	9.066
Big	917	1.293	1.332	1.327	1.095	1.468	3.328	2.406	0.357	-0.064	0.284	0.930	0.272	0.692	0.820	4.310	3.236	2.459
Meramec	3788	0.872	0.828	0.697	0.464	2.047	3.333	4.192	0.064	-0.106	-0.217	0.352	-0.302	0.282	-0.342	0.698	1.170	2.191
Mean		2.687	2.531	2.366	2.263	2.468	7.655	5.328	1.356	1.609	1.633	0.805	0.584	0.692	0.740	1.023	1.717	2.437
Standard Deviation		4.805	4.742	4.440	3.940	3.817	12.177	6.649	1.448	2.056	2.578	1.333	1.108	0.789	1.070	1.901	3.367	4.721
Yellowstone	2623	-0.355	-0.304	-0.124	-0.347	-0.402	-0.220	-0.169	-0.367	0.190	0.062	0.154	0.348	0.426	1.148	1.264	1.378	1.312
Clarks Fork	1154	-0.145	0.075	0.399	0.135	-0.027	1.256	0.109	-0.113	0.166	0.080	-0.213	-0.311	-0.486	-0.309	-0.322	-0.467	0.155
Yellowstone	11795	0.324	0.207	-0.059	-0.380	-0.256	-0.412	-0.524	-0.815	-0.097	-0.065	-0.030	0.001	-0.002	0.697	0.989	0.900	0.938
Big Sioux	8424	2.326	2.200	2.142	2.143	3.175	3.826	4.305	4.293	5.307	6.473	4.738	3.854	3.992	4.883	8.426	7.225	6.413
North Platte	1431	0.337	0.789	1.246	1.297	1.556	0.944	0.559	2.014	-0.255	-0.335	-0.466	-0.237	0.773	0.813	0.353	0.443	0.062
Bear	164	-0.657	-0.537	-0.226	-0.165	-0.314	-0.377	-0.255	1.327	0.024	0.097	0.601	0.814	0.806	1.475	1.937	3.576	3.210
Elkhorn	6900	2.589	2.089	1.522	0.987	1.321	1.591	1.193	0.829	1.387	2.140	2.532	0.828	0.237	1.594	2.692	6.863	12.187
Nishnabottna	2806	5.827	4.867	4.439	3.849	4.888	4.177	4.269	4.278	4.473	4.053	4.579	4.548	6.431	4.513	2.752	2.375	3.872
Grand	2250	6.428	5.432	5.256	7.941	6.171	14.683	10.782	1.598	5.719	6.273	6.015	8.052	13.278	8.013	6.498	3.609	3.381
Thompson	1670	13.510	12.302	10.169	6.961	7.489	14.261	10.360	0.347	1.258	1.037	2.343	2.763	7.453	6.981	5.292	3.594	2.485
Gasconade	2840	-0.045	-0.071	-0.087	0.097	1.893	1.408	1.832	0.689	0.534	-0.129	0.603	1.203	0.317	0.255	1.727	2.069	3.241
Mean		2.740	2.459	2.243	2.047	2.318	3.740	2.951	1.280	1.701	1.790	1.896	1.987	3.020	2.733	2.874	2.870	3.387
Standard Deviation		4.337	3.851	3.226	2.963	2.777	5.519	4.123	1.708	2.302	2.613	2.301	2.600	4.366	2.869	2.734	2.459	3.457

Table C-2a: Spectral Coefficient of Variation (Cv) in Log Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	0.104	0.042	0.036	0.037	0.037	0.036	0.037	0.045	0.039	0.041	0.041	0.041	0.043	0.046	0.045	0.043	0.042
Jump	576	0.132	0.116	0.114	0.115	0.121	0.119	0.116	0.122	0.058	0.050	0.052	0.049	0.050	0.051	0.054	0.057	0.058
Black	749	0.296	0.286	0.269	0.253	0.208	0.170	0.161	0.117	0.067	0.059	0.057	0.056	0.056	0.058	0.062	0.062	0.063
Maquaketa	1553	0.079	0.078	0.079	0.080	0.079	0.077	0.078	0.070	0.067	0.069	0.070	0.071	0.069	0.067	0.066	0.068	0.068
Mississippi	85600	0.031	0.032	0.032	0.032	0.031	0.031	0.033	0.033	0.030	0.031	0.032	0.032	0.032	0.033	0.033	0.033	0.032
Rock	3340	0.137	0.109	0.091	0.087	0.085	0.087	0.085	0.069	0.059	0.057	0.058	0.059	0.057	0.056	0.054	0.052	0.051
Sugar	923	0.085	0.072	0.066	0.062	0.058	0.060	0.061	0.057	0.054	0.058	0.062	0.066	0.073	0.076	0.075	0.078	0.083
Pecatonica	1326	0.088	0.078	0.075	0.073	0.070	0.070	0.070	0.063	0.058	0.061	0.061	0.064	0.066	0.065	0.063	0.064	0.065
Cedar	6510	0.087	0.085	0.085	0.085	0.086	0.089	0.094	0.085	0.078	0.078	0.077	0.076	0.076	0.074	0.073	0.074	0.074
Skunk	4303	0.229	0.220	0.220	0.212	0.206	0.192	0.191	0.136	0.099	0.093	0.091	0.091	0.086	0.081	0.074	0.069	0.063
Mississippi	119000	0.041	0.041	0.039	0.037	0.036	0.036	0.037	0.037	0.033	0.034	0.035	0.035	0.034	0.033	0.033	0.032	0.031
Des Moines	5452	0.194	0.188	0.185	0.180	0.177	0.177	0.179	0.145	0.111	0.106	0.102	0.101	0.096	0.089	0.085	0.081	0.076
Raccoon	3441	0.187	0.187	0.187	0.183	0.180	0.163	0.165	0.142	0.108	0.103	0.101	0.097	0.091	0.087	0.081	0.077	0.074
Iroquois	2091	0.185	0.176	0.164	0.164	0.172	0.194	0.184	0.112	0.071	0.063	0.059	0.058	0.056	0.054	0.053	0.051	0.051
Kankakee	2294	0.042	0.041	0.041	0.041	0.041	0.050	0.053	0.050	0.042	0.041	0.040	0.041	0.039	0.037	0.038	0.039	0.041
Spoon	1636	0.215	0.209	0.201	0.194	0.178	0.181	0.178	0.117	0.086	0.080	0.073	0.071	0.069	0.067	0.063	0.058	0.056
La Moines	1293	0.280	0.269	0.256	0.246	0.232	0.241	0.254	0.160	0.115	0.108	0.103	0.100	0.098	0.092	0.084	0.075	0.073
Meramec	781	0.055	0.054	0.054	0.054	0.054	0.064	0.072	0.090	0.076	0.077	0.075	0.074	0.075	0.080	0.084	0.088	0.088
Bourbeuse	808	0.107	0.106	0.103	0.100	0.100	0.137	0.158	0.131	0.093	0.083	0.078	0.073	0.072	0.070	0.069	0.065	0.060
Big	917	0.096	0.092	0.085	0.082	0.082	0.093	0.099	0.093	0.075	0.074	0.070	0.070	0.068	0.068	0.072	0.073	0.069
Meramec	3788	0.054	0.054	0.053	0.053	0.054	0.063	0.072	0.076	0.063	0.062	0.061	0.060	0.060	0.062	0.064	0.064	0.061
Mean		0.130	0.121	0.116	0.113	0.109	0.111	0.113	0.093	0.071	0.068	0.067	0.066	0.065	0.064	0.063	0.062	0.061
Standard Deviation		0.079	0.079	0.075	0.072	0.066	0.063	0.063	0.039	0.025	0.023	0.021	0.020	0.019	0.018	0.016	0.016	0.015
Yellowstone	2623	0.037	0.036	0.036	0.035	0.035	0.035	0.033	0.029	0.027	0.029	0.030	0.030	0.031	0.032	0.031	0.029	0.028
Clarks Fork	1154	0.086	0.085	0.081	0.079	0.074	0.051	0.034	0.033	0.032	0.033	0.031	0.030	0.031	0.032	0.030	0.027	0.027
Yellowstone	11795	0.046	0.042	0.035	0.031	0.029	0.026	0.023	0.022	0.028	0.031	0.032	0.032	0.032	0.032	0.032	0.030	0.029
Big Sioux	8424	0.246	0.244	0.238	0.232	0.222	0.205	0.197	0.154	0.131	0.127	0.125	0.122	0.119	0.117	0.114	0.112	0.110
North Platte	1431	0.107	0.107	0.104	0.100	0.091	0.085	0.082	0.075	0.077	0.080	0.080	0.081	0.077	0.075	0.070	0.067	0.065
Bear	164	0.318	0.261	0.217	0.198	0.176	0.150	0.133	0.123	0.136	0.146	0.154	0.155	0.154	0.155	0.154	0.150	0.144
Elkhorn	6900	0.107	0.106	0.106	0.104	0.097	0.088	0.082	0.082	0.079	0.082	0.087	0.087	0.092	0.095	0.095	0.093	0.086
Nishnabottna	2806	0.238	0.228	0.219	0.204	0.189	0.169	0.158	0.133	0.103	0.099	0.100	0.097	0.094	0.090	0.087	0.083	0.079
Grand	2250	0.356	0.340	0.323	0.312	0.292	0.276	0.273	0.212	0.117	0.106	0.102	0.097	0.094	0.088	0.080	0.071	0.060
Thompson	1670	0.343	0.319	0.301	0.281	0.265	0.247	0.247	0.201	0.112	0.101	0.097	0.093	0.086	0.080	0.075	0.072	0.069
Gasconade	2840	0.041	0.041	0.041	0.042	0.044	0.049	0.055	0.073	0.062	0.064	0.061	0.061	0.062	0.063	0.064	0.067	0.066
Mean		0.175	0.164	0.155	0.147	0.138	0.126	0.120	0.103	0.082	0.082	0.082	0.080	0.079	0.078	0.076	0.073	0.069
Standard Deviation		0.127	0.116	0.108	0.102	0.095	0.089	0.088	0.067	0.041	0.039	0.040	0.040	0.039	0.038	0.038	0.037	0.036

Table C-2b: Spectral Coefficient of Skewness (Sk) in Log Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	-1.355	-0.215	-0.306	-0.433	-0.429	-0.215	0.007	0.347	-0.530	-0.582	-0.646	-0.609	-0.467	-0.500	-0.680	-0.711	-0.633
Jump	576	-0.593	-0.402	-0.368	-0.257	-0.010	0.074	0.088	-0.358	-0.488	-0.391	-0.276	-0.200	-0.008	0.108	0.103	-0.011	-0.062
Black	749	-1.337	-1.356	-1.286	-1.294	-0.663	-0.208	-0.256	-0.582	-0.626	-0.894	-0.946	-1.362	-1.035	-0.934	-0.931	-0.865	-0.644
Maquaketa	1553	-0.074	-0.211	-0.249	-0.301	-0.402	-0.272	-0.164	-0.342	-0.374	-0.422	-0.536	-0.375	-0.299	-0.148	-0.107	-0.137	-0.147
Mississippi	85600	-0.861	-0.843	-0.570	-0.363	-0.127	-0.031	0.026	-0.017	-0.584	-0.697	-0.828	-0.801	-0.548	-0.531	-0.566	-0.615	-0.608
Rock	3340	-0.265	-0.307	0.088	0.116	0.106	0.047	0.075	-0.335	-0.526	-0.659	-0.669	-0.590	-0.561	-0.624	-0.621	-0.578	-0.598
Sugar	923	-0.320	-0.248	-0.105	0.026	-0.015	0.081	0.078	0.105	-0.183	-0.260	-0.341	-0.374	-0.309	-0.202	-0.293	-0.209	-0.093
Pecatonica	1326	0.091	0.166	0.175	0.196	0.192	0.208	0.171	0.096	-0.180	-0.296	-0.451	-0.419	-0.298	-0.148	-0.079	0.086	0.144
Cedar	6510	-0.110	-0.016	-0.093	-0.060	-0.083	-0.024	0.091	-0.418	-0.466	-0.538	-0.626	-0.717	-0.808	-0.723	-0.692	-0.638	-0.640
Skunk	4303	-0.482	-0.488	-0.493	-0.441	-0.469	-0.394	-0.599	-1.088	-1.146	-1.294	-1.393	-1.418	-1.606	-1.598	-1.390	-1.198	-1.013
Mississippi	119000	-0.610	-0.522	-0.337	-0.215	-0.089	-0.038	-0.031	-0.284	-0.524	-0.631	-0.824	-0.847	-0.767	-0.854	-0.918	-0.828	-0.717
Des Moines	5452	0.014	-0.015	-0.035	-0.003	0.027	-0.001	0.068	-0.441	-0.669	-0.711	-0.876	-0.951	-0.914	-0.825	-0.838	-0.806	-0.741
Raccoon	3441	-0.145	-0.112	-0.135	-0.161	-0.011	0.060	0.236	-0.399	-0.537	-0.571	-0.698	-0.699	-0.737	-0.744	-0.682	-0.510	-0.392
Iroquois	2091	0.056	0.022	0.047	0.063	0.371	0.550	-0.024	-1.097	-0.946	-1.131	-1.041	-0.908	-0.816	-0.770	-0.572	-0.492	-0.452
Kankakee	2294	0.023	0.017	0.006	0.076	0.202	0.520	0.285	-0.507	-0.618	-0.830	-0.924	-0.805	-0.911	-0.850	-0.770	-0.625	-0.583
Spoon	1636	0.526	0.491	0.475	0.432	0.240	0.473	0.222	-0.249	-0.826	-1.061	-1.082	-0.855	-0.750	-0.640	-0.529	-0.503	-0.272
La Moines	1293	-0.759	-0.545	-0.318	-0.135	-0.060	-0.017	-0.041	-0.675	-1.239	-1.431	-1.487	-1.441	-1.252	-1.280	-1.231	-1.061	-0.871
Meramec	781	-0.009	0.033	0.069	0.060	0.163	0.720	0.558	-0.002	-0.194	-0.281	-0.323	-0.419	-0.457	-0.479	-0.518	-0.668	-0.940
Bourbeuse	808	-0.056	-0.032	-0.037	-0.052	-0.088	1.144	0.779	-0.447	-0.563	-0.665	-0.547	-0.547	-0.543	-0.439	-0.364	-0.287	0.024
Big	917	-0.562	-0.567	-0.400	-0.414	-0.255	0.088	0.408	-0.390	-0.443	-0.548	-0.650	-0.605	-0.558	-0.567	-0.557	-0.561	-0.552
Meramec	3788	-0.271	-0.290	-0.278	-0.295	-0.054	0.410	0.682	-0.209	-0.433	-0.551	-0.583	-0.613	-0.619	-0.654	-0.570	-0.583	-0.378
Mean		-0.338	-0.259	-0.198	-0.164	-0.069	0.151	0.127	-0.347	-0.576	-0.688	-0.750	-0.741	-0.679	-0.638	-0.610	-0.562	-0.484
Standard Deviation		0.469	0.390	0.351	0.345	0.258	0.368	0.309	0.352	0.278	0.322	0.322	0.341	0.355	0.382	0.355	0.319	0.319
Yellowstone	2623	-0.175	-0.116	-0.070	-0.104	-0.173	-0.201	-0.253	-0.299	-0.078	-0.185	-0.202	-0.142	-0.102	-0.030	-0.067	-0.123	-0.307
Clarks Fork	1154	-1.123	-1.204	-1.451	-1.466	-1.510	-2.060	-0.809	-0.263	-0.020	-0.127	-0.179	-0.099	-0.077	0.038	0.002	-0.081	-0.046
Yellowstone	11795	-0.750	-0.598	-0.238	-0.279	-0.541	-0.458	-0.242	-0.117	-0.155	-0.247	-0.320	-0.262	-0.209	-0.150	-0.177	-0.308	-0.409
Big Sioux	8424	0.116	0.101	0.145	0.141	0.153	0.372	0.502	-0.192	0.011	-0.165	-0.306	-0.402	-0.438	-0.372	-0.373	-0.357	-0.318
North Platte	1431	0.171	0.190	0.153	0.051	-0.088	-0.108	-0.197	0.557	-0.363	-0.474	-0.557	-0.570	-0.609	-0.688	-0.636	-0.609	-0.628
Bear	164	-1.094	-0.706	-0.565	-0.478	-0.462	-0.421	-0.297	0.187	0.019	-0.001	-0.016	0.056	0.079	0.114	0.161	0.304	0.437
Elkhorn	6900	0.245	0.100	-0.005	-0.099	-0.094	0.180	0.442	0.234	0.190	0.125	0.062	-0.064	-0.162	-0.105	-0.015	0.036	-0.022
Nishnabottna	2806	-0.653	-0.567	-0.509	-0.473	-0.474	-0.353	-0.149	-0.575	-0.344	-0.464	-0.591	-0.620	-0.771	-0.848	-0.973	-1.165	-1.273
Grand	2250	-0.440	-0.387	-0.383	-0.371	-0.404	-0.152	-0.340	-1.049	-0.395	-0.452	-0.380	-0.312	-0.311	-0.569	-0.444	-0.427	-0.479
Thompson	1670	-0.758	-0.665	-0.621	-0.514	-0.424	0.054	-0.257	-1.065	-0.495	-0.575	-0.487	-0.488	-0.400	-0.806	-0.586	-0.275	-0.285
Gasconade	2840	-0.187	-0.196	-0.220	-0.216	-0.139	0.094	0.200	-0.322	-0.497	-0.615	-0.550	-0.533	-0.556	-0.534	-0.526	-0.543	-0.477
Mean		-0.422	-0.368	-0.342	-0.346	-0.378	-0.278	-0.127	-0.264	-0.193	-0.289	-0.321	-0.312	-0.323	-0.359	-0.330	-0.323	-0.346
Standard Deviation		0.492	0.429	0.456	0.430	0.433	0.645	0.376	0.501	0.235	0.243	0.221	0.229	0.258	0.349	0.342	0.384	0.423

Table C-2c: Spectral Coefficient of Kurtosis (Ku) in Log Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	1.338	-0.344	0.122	0.142	0.148	-0.108	-0.054	-0.877	-0.119	-0.048	0.073	0.190	-0.204	-0.196	0.340	0.431	0.236
Jump	576	-0.036	-0.344	-0.304	-0.347	0.248	-0.039	-0.682	-0.690	-0.513	-0.388	-0.494	-0.542	-0.509	-0.151	0.328	1.010	1.461
Black	749	2.396	2.776	2.366	2.697	0.291	-0.375	-0.224	0.025	0.440	1.420	1.623	2.977	1.630	1.418	1.279	0.982	0.877
Maquaketa	1553	-0.501	-0.447	-0.431	-0.480	-0.594	-0.777	-0.846	-0.053	0.148	0.199	0.313	-0.067	-0.150	-0.347	-0.761	-1.016	-0.981
Mississippi	85600	0.749	0.772	0.148	-0.214	-0.531	-0.728	-0.678	-0.431	0.092	0.625	0.749	0.719	0.461	0.572	0.601	0.724	0.752
Rock	3340	-0.275	0.292	-0.504	-0.628	-0.678	-0.813	-0.810	-0.369	0.149	0.345	0.303	0.175	0.534	0.865	0.928	0.830	0.731
Sugar	923	-0.537	-0.499	-0.735	-0.886	-0.784	-0.812	-0.916	-0.340	-0.196	-0.241	-0.496	-0.367	-0.326	-0.397	-0.444	-0.527	-0.575
Pecatonica	1326	-1.086	-0.983	-1.030	-0.997	-0.981	-0.971	-0.903	-0.279	0.155	0.142	-0.067	-0.130	-0.338	-0.533	-0.796	-0.799	-0.787
Cedar	6510	-0.447	-0.836	-0.858	-0.854	-0.926	-0.914	-0.605	-0.279	0.141	0.511	0.335	0.476	0.376	0.202	-0.028	-0.053	0.024
Skunk	4303	-0.360	-0.162	-0.211	-0.452	-0.489	-0.096	-0.162	0.904	2.151	2.706	3.109	3.199	4.183	3.709	3.275	2.787	2.305
Mississippi	119000	0.633	0.214	-0.423	-0.585	-0.678	-0.820	-0.897	-0.221	0.588	1.183	1.149	1.318	1.459	1.520	1.693	1.288	0.988
Des Moines	5452	-0.548	-0.485	-0.430	-0.303	-0.279	-0.545	-0.540	-0.450	0.322	0.654	1.035	1.196	0.795	0.493	0.641	0.753	0.862
Raccoon	3441	0.455	0.512	0.474	0.491	0.459	0.055	-0.008	-0.305	0.209	0.268	0.488	0.469	0.509	0.468	0.176	-0.055	-0.077
Iroquois	2091	-0.258	-0.182	-0.341	-0.487	-0.649	0.145	-0.989	0.966	0.959	1.271	1.159	1.075	0.940	0.541	-0.028	-0.069	-0.057
Kankakee	2294	1.093	1.258	1.016	0.780	0.434	0.478	0.000	-0.083	0.279	0.686	0.980	0.850	1.180	1.109	0.950	0.873	0.835
Spoon	1636	0.818	0.794	0.749	0.394	-0.102	0.206	-0.462	-0.120	1.205	1.598	1.607	0.796	0.262	-0.302	-0.457	-0.353	-0.471
La Moines	1293	2.493	1.700	1.008	0.275	-0.135	-0.567	-0.467	-0.057	1.915	2.423	2.975	2.912	2.207	2.559	2.372	2.260	1.386
Meramec	781	-0.735	-0.718	-0.679	-0.618	-0.190	0.747	-0.319	-0.552	-0.635	-0.545	-0.413	-0.210	0.312	0.270	0.488	0.854	1.465
Bourbeuse	808	-0.473	-0.540	-0.482	-0.381	-0.249	1.877	0.238	-0.288	-0.037	0.026	-0.298	-0.272	-0.276	-0.333	-0.030	0.018	0.175
Big	917	0.186	0.333	0.085	0.149	0.332	0.061	0.112	-0.422	-0.445	-0.306	-0.180	-0.296	-0.311	0.082	0.300	0.226	0.389
Meramec	3788	-0.050	0.004	-0.030	-0.053	0.336	0.109	0.239	-0.617	-0.411	-0.279	-0.371	-0.321	-0.110	0.041	-0.028	0.281	0.077
Mean		0.231	0.148	-0.023	-0.112	-0.239	-0.185	-0.427	-0.216	0.305	0.583	0.647	0.674	0.601	0.552	0.514	0.497	0.458
Standard Deviation		0.973	0.919	0.792	0.801	0.471	0.684	0.406	0.443	0.730	0.897	1.041	1.129	1.102	1.054	1.001	0.922	0.830
Yellowstone	2623	-0.494	-0.493	-0.400	-0.452	-0.342	-0.164	-0.017	0.045	-0.387	-0.345	-0.267	-0.140	-0.018	0.269	0.377	0.628	0.928
Clarks Fork	1154	1.240	1.493	2.444	2.314	2.557	6.666	0.825	0.071	-0.116	-0.097	-0.218	-0.335	-0.504	-0.637	-0.584	-0.407	-0.228
Yellowstone	11795	1.132	0.832	-0.102	-0.356	0.180	-0.133	-0.407	-0.785	-0.599	-0.489	-0.424	-0.363	-0.114	0.142	0.312	0.617	0.798
Big Sioux	8424	0.131	0.142	0.077	0.082	0.164	-0.320	-0.344	-0.632	-0.472	-0.145	0.159	0.364	0.507	0.433	0.320	0.067	-0.005
North Platte	1431	-0.465	-0.351	-0.167	0.053	0.479	0.191	0.319	0.220	-0.217	-0.171	-0.121	-0.076	0.128	0.325	0.093	0.119	0.044
Bear	164	1.155	-0.328	-0.379	-0.430	-0.290	-0.054	-0.012	-0.464	-0.598	-0.623	-0.521	-0.609	-0.736	-0.781	-0.865	-0.871	-0.756
Elkhorn	6900	-0.081	-0.158	-0.253	-0.224	-0.077	-0.110	-0.150	-0.768	-0.738	-0.705	-0.668	-0.765	-0.812	-0.606	-0.607	-0.577	-0.419
Nishnabottna	2806	0.100	-0.171	-0.301	-0.320	-0.046	-0.046	-0.112	0.122	-0.152	0.162	0.688	0.816	1.416	1.285	1.433	2.345	3.281
Grand	2250	-0.369	-0.400	-0.327	-0.222	-0.108	-0.249	-0.381	0.886	-0.431	-0.229	-0.033	0.096	0.389	0.413	0.139	-0.098	0.320
Thompson	1670	0.560	0.346	0.319	0.268	0.196	0.348	-0.453	0.721	-0.394	-0.164	0.078	0.262	0.586	2.027	1.254	0.161	-0.531
Gasconade	2840	-0.385	-0.407	-0.407	-0.392	0.128	-0.171	0.022	-0.347	0.058	0.229	0.480	0.636	0.604	0.334	0.383	0.376	0.452
Mean		0.229	0.046	0.046	0.029	0.258	0.542	-0.064	-0.085	-0.368	-0.234	-0.077	-0.010	0.131	0.291	0.205	0.215	0.353
Standard Deviation		0.683	0.624	0.826	0.794	0.798	2.040	0.375	0.569	0.239	0.293	0.412	0.500	0.665	0.829	0.717	0.850	1.108

Table C-3a: Spectral Coefficient of Variation (Cv) in Real Space via Log Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	0.704	0.675	0.651	0.633	0.667	0.707	0.782	0.653	0.517	0.506	0.516	0.517	0.545	0.566	0.559	0.576	0.584
Jump	576	0.362	0.340	0.325	0.327	0.349	0.378	0.427	0.500	0.385	0.393	0.410	0.419	0.445	0.475	0.479	0.501	0.515
Black	749	0.329	0.307	0.293	0.292	0.320	0.351	0.423	0.512	0.395	0.402	0.421	0.430	0.459	0.491	0.495	0.520	0.537
Maquaketa	1553	0.561	0.532	0.511	0.499	0.528	0.560	0.621	0.539	0.425	0.422	0.431	0.433	0.459	0.485	0.490	0.515	0.537
Mississippi	85600	0.948	0.896	0.857	0.832	0.892	0.960	1.074	0.832	0.661	0.634	0.632	0.630	0.657	0.680	0.654	0.674	0.686
Rock	3340	0.562	0.537	0.525	0.517	0.547	0.590	0.660	0.584	0.458	0.455	0.462	0.465	0.484	0.498	0.487	0.495	0.499
Sugar	923	0.558	0.525	0.504	0.496	0.521	0.546	0.607	0.527	0.417	0.413	0.422	0.425	0.448	0.475	0.474	0.488	0.491
Pecatonica	1326	0.490	0.464	0.444	0.431	0.451	0.470	0.520	0.453	0.360	0.357	0.369	0.371	0.388	0.415	0.425	0.448	0.454
Cedar	6510	0.634	0.602	0.582	0.568	0.598	0.638	0.718	0.635	0.501	0.493	0.497	0.502	0.533	0.556	0.552	0.573	0.582
Skunk	4303	0.483	0.452	0.433	0.427	0.470	0.523	0.623	0.612	0.474	0.476	0.486	0.494	0.521	0.547	0.541	0.559	0.567
Mississippi	119000	0.966	0.917	0.882	0.852	0.919	0.993	1.120	0.858	0.679	0.653	0.648	0.645	0.672	0.696	0.669	0.690	0.703
Des Moines	5452	0.480	0.455	0.437	0.425	0.458	0.494	0.562	0.597	0.463	0.465	0.475	0.478	0.510	0.536	0.527	0.547	0.554
Raccoon	3441	0.463	0.433	0.421	0.417	0.449	0.487	0.555	0.569	0.439	0.444	0.455	0.458	0.489	0.515	0.518	0.532	0.543
Iroquois	2091	0.629	0.591	0.565	0.551	0.580	0.615	0.690	0.594	0.469	0.463	0.467	0.467	0.409	0.500	0.491	0.502	0.506
Kankakee	2294	0.384	0.365	0.352	0.352	0.383	0.445	0.563	0.590	0.456	0.460	0.471	0.473	0.410	0.522	0.520	0.538	0.545
Spoon	1636	0.949	0.345	0.336	0.338	0.371	0.434	0.523	0.556	0.427	0.435	0.445	0.450	0.487	0.511	0.513	0.534	0.542
La Moines	1293	0.300	0.279	0.272	0.271	0.320	0.362	0.448	0.538	0.412	0.422	0.432	0.440	0.493	0.500	0.504	0.520	0.528
Meramec	781	0.487	0.455	0.434	0.422	0.441	0.466	0.517	0.507	0.393	0.398	0.406	0.410	0.478	0.466	0.473	0.509	0.542
Bourbeuse	808	0.344	0.323	0.310	0.304	0.327	0.356	0.425	0.522	0.398	0.409	0.423	0.426	0.426	0.488	0.497	0.532	0.545
Big	917	0.452	0.424	0.408	0.401	0.423	0.462	0.525	0.541	0.419	0.424	0.433	0.440	0.470	0.491	0.500	0.534	0.553
Meramec	3788	0.604	0.567	0.542	0.526	0.555	0.601	0.680	0.632	0.493	0.491	0.500	0.505	0.438	0.562	0.565	0.593	0.607
Mean		0.557	0.499	0.480	0.471	0.503	0.545	0.622	0.588	0.459	0.458	0.467	0.470	0.487	0.523	0.521	0.542	0.553
Standard Deviation		0.198	0.172	0.165	0.157	0.165	0.173	0.187	0.099	0.081	0.072	0.068	0.066	0.072	0.066	0.057	0.057	0.058
Yellowstone	2623	0.789	0.774	0.772	0.742	0.701	0.652	0.639	0.670	0.659	0.673	0.758	0.757	0.690	0.699	0.648	0.623	0.613
Clarks Fork	1154	0.588	0.578	0.573	0.563	0.537	0.512	0.507	0.524	0.556	0.579	0.661	0.668	0.616	0.626	0.586	0.567	0.557
Yellowstone	11795	0.873	0.867	0.880	0.861	0.830	0.775	0.756	0.794	0.733	0.739	0.832	0.827	0.752	0.769	0.707	0.680	0.671
Big Sioux	8424	0.496	0.484	0.477	0.461	0.445	0.436	0.452	0.600	0.547	0.571	0.638	0.648	0.607	0.628	0.597	0.589	0.582
North Platte	1431	0.468	0.461	0.458	0.449	0.427	0.409	0.405	0.437	0.486	0.511	0.579	0.583	0.539	0.547	0.516	0.503	0.494
Bear	164	0.265	0.264	0.265	0.261	0.247	0.235	0.236	0.264	0.305	0.332	0.374	0.384	0.362	0.368	0.347	0.338	0.334
Elkhorn	6900	0.660	0.648	0.653	0.637	0.615	0.581	0.577	0.654	0.571	0.580	0.634	0.639	0.600	0.624	0.597	0.594	0.596
Nishnabottna	2806	0.588	0.587	0.589	0.572	0.550	0.539	0.539	0.633	0.573	0.583	0.642	0.644	0.608	0.628	0.608	0.601	0.603
Grand	2250	0.385	0.373	0.376	0.380	0.392	0.415	0.451	0.601	0.570	0.595	0.657	0.661	0.624	0.661	0.632	0.637	0.641
Thompson	1670	0.422	0.411	0.412	0.407	0.414	0.418	0.470	0.591	0.552	0.573	0.636	0.637	0.605	0.634	0.612	0.616	0.622
Gasconade	2840	0.750	0.732	0.719	0.697	0.664	0.632	0.634	0.733	0.642	0.655	0.713	0.710	0.661	0.692	0.660	0.652	0.649
Mean		0.571	0.562	0.561	0.548	0.529	0.509	0.515	0.591	0.563	0.581	0.648	0.651	0.606	0.625	0.592	0.582	0.578
Standard Deviation		0.186	0.184	0.185	0.177	0.166	0.148	0.139	0.144	0.108	0.103	0.114	0.111	0.098	0.102	0.094	0.093	0.094

Table C-3b: Spectral Coefficient of Skewness (Sk) in Real Space via Log Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	1.127	1.132	1.076	1.008	1.281	1.646	1.859	1.335	0.955	0.806	0.693	0.666	0.815	1.005	0.768	0.981	1.168
Jump	576	0.463	0.477	0.483	0.473	0.650	0.855	1.079	0.912	0.562	0.467	0.409	0.418	0.554	0.749	0.578	0.800	0.997
Black	749	0.394	0.411	0.426	0.410	0.592	0.795	1.070	0.942	0.593	0.492	0.440	0.446	0.590	0.791	0.615	0.845	1.052
Maquaketa	1553	0.854	0.856	0.820	0.773	1.007	1.277	1.504	1.017	0.680	0.554	0.465	0.454	0.590	0.774	0.604	0.831	1.052
Mississippi	85600	1.582	1.563	1.464	1.369	1.724	2.137	2.502	1.847	1.380	1.185	1.001	0.941	1.110	1.333	0.989	1.213	1.418
Rock	3340	0.857	0.866	0.846	0.803	1.043	1.351	1.590	1.141	0.780	0.654	0.550	0.535	0.656	0.812	0.596	0.783	0.955
Sugar	923	0.848	0.841	0.806	0.767	0.993	1.245	1.472	0.984	0.656	0.527	0.441	0.433	0.559	0.746	0.566	0.766	0.935
Pecatonica	1326	0.717	0.723	0.697	0.653	0.853	1.065	1.280	0.783	0.489	0.357	0.299	0.297	0.404	0.580	0.448	0.670	0.841
Cedar	6510	0.995	0.990	0.948	0.893	1.145	1.472	1.718	1.285	0.908	0.768	0.643	0.627	0.785	0.979	0.751	0.972	1.164
Skunk	4303	0.703	0.700	0.679	0.647	0.891	1.190	1.509	1.222	0.827	0.718	0.615	0.607	0.752	0.952	0.724	0.940	1.127
Mississippi	119000	1.616	1.605	1.512	1.407	1.776	2.409	2.602	1.923	1.435	1.239	1.042	0.978	1.147	1.381	1.025	1.252	1.460
Des Moines	5452	0.697	0.706	0.685	0.644	0.867	1.122	1.374	1.177	0.792	0.683	0.586	0.567	0.723	0.920	0.692	0.910	1.095
Raccoon	3441	0.663	0.663	0.656	0.629	0.850	1.103	1.357	1.101	0.724	0.622	0.532	0.518	0.669	0.860	0.671	0.874	1.065
Iroquois	2091	0.985	0.969	0.917	0.863	1.110	1.412	1.655	1.171	0.811	0.678	0.563	0.541	0.457	0.819	0.607	0.801	0.973
Kankakee	2294	0.507	0.529	0.531	0.515	0.718	1.007	1.376	1.158	0.774	0.667	0.573	0.555	0.461	0.881	0.675	0.889	1.071
Spoon	1636	1.579	0.489	0.502	0.491	0.694	0.982	1.287	1.065	0.686	0.593	0.503	0.497	0.663	0.849	0.658	0.878	1.064
La Moines	1293	0.335	0.354	0.386	0.374	0.591	0.819	1.125	1.016	0.644	0.553	0.469	0.471	0.678	0.818	0.636	0.846	1.030
Meramec	781	0.711	0.705	0.680	0.639	0.834	1.056	1.273	0.928	0.586	0.480	0.399	0.397	0.639	0.723	0.563	0.819	1.064
Bourbeuse	808	0.427	0.444	0.455	0.432	0.605	0.806	1.074	0.970	0.602	0.515	0.445	0.438	0.502	0.785	0.619	0.873	1.071
Big	917	0.643	0.645	0.633	0.601	0.798	1.046	1.292	1.024	0.662	0.559	0.473	0.471	0.617	0.794	0.628	0.879	1.091
Meramec	3788	0.938	0.923	0.876	0.822	1.059	1.379	1.634	1.278	0.882	0.762	0.652	0.635	0.535	0.995	0.781	1.022	1.225
Mean		0.840	0.790	0.766	0.724	0.956	1.246	1.506	1.156	0.782	0.661	0.562	0.547	0.662	0.883	0.676	0.897	1.091
Standard Deviation		0.378	0.336	0.303	0.278	0.326	0.414	0.410	0.280	0.240	0.215	0.180	0.163	0.189	0.187	0.134	0.137	0.143
Yellowstone	2623	1.467	1.582	1.703	1.600	1.638	1.477	1.393	1.588	1.440	1.547	1.858	1.694	1.540	1.973	1.519	1.372	1.505
Clarks Fork	1154	1.141	1.218	1.287	1.241	1.299	1.232	1.190	1.304	1.184	1.252	1.569	1.462	1.325	1.708	1.331	1.205	1.323
Yellowstone	11795	1.595	1.743	1.914	1.820	1.875	1.669	1.549	1.825	1.624	1.760	2.074	1.871	1.718	2.218	1.695	1.539	1.691
Big Sioux	8424	0.979	1.028	1.074	1.015	1.086	1.084	1.096	1.453	1.163	1.227	1.500	1.411	1.299	1.717	1.364	1.270	1.405
North Platte	1431	0.930	0.981	1.031	0.988	1.043	1.027	1.012	1.131	1.012	1.044	1.325	1.241	1.097	1.416	1.113	1.016	1.119
Bear	164	0.535	0.544	0.567	0.532	0.571	0.632	0.662	0.783	0.569	0.526	0.723	0.716	0.574	0.743	0.583	0.527	0.603
Elkhorn	6900	1.261	1.352	1.459	1.394	1.466	1.357	1.301	1.557	1.222	1.254	1.489	1.387	1.279	1.703	1.365	1.286	1.452
Nishnabottna	2806	1.141	1.236	1.322	1.260	1.329	1.281	1.241	1.517	1.227	1.265	1.512	1.400	1.303	1.717	1.398	1.307	1.474
Grand	2250	0.775	0.792	0.839	0.827	0.956	1.041	1.096	1.454	1.220	1.300	1.558	1.443	1.348	1.836	1.470	1.414	1.596
Thompson	1670	0.844	0.874	0.923	0.891	1.010	1.047	1.127	1.435	1.176	1.234	1.494	1.383	1.294	1.738	1.409	1.352	1.536
Gasconade	2840	1.405	1.507	1.597	1.514	1.567	1.445	1.385	1.708	1.398	1.488	1.724	1.571	1.456	1.948	1.555	1.458	1.620
Mean		1.098	1.169	1.247	1.189	1.258	1.208	1.187	1.432	1.203	1.263	1.530	1.416	1.294	1.702	1.346	1.250	1.393
Standard Deviation		0.322	0.365	0.405	0.380	0.370	0.283	0.236	0.284	0.267	0.312	0.338	0.289	0.288	0.377	0.292	0.276	0.305

Table C-3c: Spectral Coefficient of Kurtosis (Ku) in Real Space via Log Space

RIVER	AREA	1-DL	3-DL	7-DL	14-DL	30-DL	60-DL	90-DL	180-DL	AM	180-DH	90-DH	60-DH	30-DH	14-DH	7-DH	3-DH	1-DH
St. Croix	6240	0.867	0.984	0.771	0.509	1.527	3.816	4.350	2.565	1.329	0.860	0.333	0.122	0.616	1.387	0.257	0.881	1.549
Jump	576	-0.422	-0.396	-0.497	-0.557	-0.086	0.607	1.117	1.088	0.423	0.256	-0.095	-0.204	0.104	0.694	-0.067	0.459	1.064
Black	749	-0.484	-0.461	-0.564	-0.623	-0.179	0.438	1.088	1.179	0.479	0.291	-0.058	-0.175	0.164	0.797	-0.010	0.557	1.212
Maquaketa	1553	0.182	0.240	0.093	-0.068	0.697	2.104	2.688	1.414	0.653	0.384	-0.027	-0.166	0.164	0.756	-0.028	0.527	1.212
Mississippi	85600	2.490	2.593	2.170	1.726	3.268	7.780	8.117	4.928	2.752	1.863	1.032	0.660	1.417	2.522	0.747	1.543	2.384
Rock	3340	0.189	0.263	0.150	-0.003	0.795	2.417	3.063	1.833	0.875	0.554	0.092	-0.069	0.282	0.848	-0.040	0.425	0.958
Sugar	923	0.171	0.208	0.061	-0.081	0.658	1.970	2.554	1.309	0.603	0.342	-0.057	-0.188	0.113	0.688	-0.084	0.390	0.909
Pecatonica	1326	-0.079	-0.033	-0.162	-0.297	0.317	1.284	1.804	0.730	0.298	0.122	-0.205	-0.307	-0.105	0.334	-0.237	0.209	0.691
Cedar	6510	0.508	0.574	0.407	0.205	1.091	2.963	3.651	2.369	1.199	0.777	0.243	0.061	0.546	1.308	0.225	0.857	1.535
Skunk	4303	-0.103	-0.076	-0.197	-0.306	0.403	1.752	2.707	2.126	0.988	0.675	0.195	0.031	0.475	1.229	0.175	0.778	1.424
Mississippi	119000	2.632	2.774	2.374	1.864	3.504	8.392	8.782	5.328	2.968	2.033	1.143	0.747	1.535	2.712	0.837	1.667	2.543
Des Moines	5452	-0.112	-0.064	-0.185	-0.312	0.349	1.491	2.158	1.963	0.905	0.608	0.147	-0.027	0.414	1.137	0.117	0.706	1.332
Raccoon	3441	-0.167	-0.140	-0.237	-0.336	0.309	1.423	2.093	1.691	0.747	0.496	0.064	-0.091	0.307	0.974	0.080	0.623	1.248
Iroquois	2091	0.483	0.517	0.326	0.134	0.986	2.688	3.357	1.940	0.949	0.598	0.112	-0.062	-0.038	0.866	-0.023	0.462	1.003
Kankakee	2294	-0.376	-0.336	-0.434	-0.503	0.037	1.085	2.165	1.891	0.861	0.578	0.127	-0.042	-0.033	1.031	0.087	0.657	1.266
Spoon	1636	2.483	-0.383	-0.473	-0.534	-0.008	1.000	1.831	1.570	0.665	0.446	0.024	-0.117	0.294	0.944	0.058	0.630	1.246
La Moines	1293	-0.527	-0.507	-0.604	-0.656	-0.181	0.503	1.264	1.409	0.578	0.382	-0.022	-0.147	0.324	0.865	0.023	0.558	1.152
Meramec	781	-0.090	-0.067	-0.194	-0.321	0.274	1.254	1.777	1.138	0.467	0.275	-0.106	-0.225	0.251	0.635	-0.088	0.500	1.245
Bourbeuse	808	-0.457	-0.431	-0.531	-0.601	-0.158	0.467	1.102	1.265	0.496	0.325	-0.052	-0.183	0.025	0.781	-0.004	0.618	1.264
Big	917	-0.198	-0.168	-0.277	-0.382	0.196	1.219	1.848	1.434	0.615	0.392	-0.017	-0.147	0.211	0.803	0.010	0.634	1.321
Meramec	3788	0.370	0.400	0.222	0.039	0.840	2.539	3.258	2.339	1.130	0.765	0.259	0.072	0.074	1.356	0.282	0.987	1.726
Mean		0.350	0.261	0.106	-0.053	0.697	2.247	2.894	1.977	0.951	0.620	0.149	-0.022	0.340	1.079	0.110	0.698	1.347
Standard Deviation		0.983	0.895	0.805	0.685	1.005	2.137	2.048	1.150	0.690	0.481	0.340	0.264	0.423	0.573	0.259	0.350	0.437
Yellowstone	2623	1.720	2.377	3.181	2.580	2.706	1.787	1.252	2.694	2.660	3.943	5.073	3.806	3.319	6.197	3.310	2.728	3.571
Clarks Fork	1154	0.762	1.222	1.681	1.432	1.674	1.142	0.796	1.630	1.694	2.591	3.689	2.808	2.512	4.912	2.624	2.134	2.850
Yellowstone	11795	2.177	2.994	4.112	3.448	3.583	2.406	1.674	3.728	3.464	5.056	6.214	4.651	4.055	7.498	4.010	3.382	4.375
Big Sioux	8424	0.395	0.749	1.081	0.875	1.156	0.826	0.619	2.165	1.621	2.486	3.385	2.604	2.419	4.953	2.740	2.358	3.165
North Platte	1431	0.295	0.647	0.976	0.817	1.064	0.719	0.479	1.081	1.141	1.780	2.666	1.980	1.755	3.650	1.923	1.538	2.129
Bear	164	-0.257	-0.040	0.125	0.099	0.323	0.179	0.069	0.203	0.120	0.329	0.763	0.524	0.463	1.441	0.633	0.392	0.740
Elkhorn	6900	1.082	1.610	2.247	1.881	2.149	1.451	1.034	2.568	1.827	2.601	3.337	2.513	2.351	4.887	2.744	2.415	3.352
Nishnabottna	2806	0.764	1.271	1.790	1.485	1.752	1.260	0.902	2.411	1.845	2.646	3.436	2.561	2.433	4.952	2.862	2.489	3.443
Grand	2250	0.030	0.291	0.557	0.507	0.892	0.744	0.618	2.169	1.820	2.795	3.642	2.733	2.594	5.516	3.123	2.886	3.955
Thompson	1670	0.141	0.435	0.728	0.622	0.998	0.755	0.676	2.098	1.664	2.518	3.361	2.498	2.401	5.049	2.900	2.653	3.697
Gasconade	2840	1.519	2.114	2.756	2.275	2.466	1.693	1.232	3.203	2.489	3.658	4.420	3.258	2.994	6.070	3.447	3.058	4.058
Mean		0.784	1.243	1.749	1.456	1.706	1.178	0.850	2.177	1.850	2.764	3.635	2.721	2.481	5.011	2.756	2.367	3.212
Standard Deviation		0.768	0.951	1.227	1.013	0.953	0.627	0.437	0.966	0.853	1.206	1.371	1.035	0.899	1.536	0.880	0.815	1.023

Description of Regional Sequences

Table C-4a: Spectral Regional and At-Site Statistical Characteristics in Real Space–Coefficient of Variation (Cv)

	Upper Mississippi Basin				Missouri Basin			
Spectral	Regional		At-Site		Regional		At-Site	
Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.467	N.A.	0.576	0.271	0.509	N.A.	0.621	0.351
3-DL	0.443	N.A.	0.551	0.275	0.497	N.A.	0.610	0.347
7-DL	0.446	N.A.	0.540	0.271	0.497	N.A.	0.596	0.347
14-DL	0.430	N.A.	0.538	0.267	0.477	N.A.	0.591	0.353
30-DL	0.440	N.A.	0.551	0.270	0.453	N.A.	0.592	0.369
60-DL	0.536	N.A.	0.675	0.373	0.402	N.A.	0.644	0.474
90-DL	0.574	N.A.	0.693	0.335	0.403	N.A.	0.617	0.437
180-DL	0.537	N.A.	0.561	0.167	0.509	N.A.	0.527	0.288
AM	0.436	N.A.	0.461	0.133	0.516	N.A.	0.517	0.235
180-DH	0.429	N.A.	0.464	0.134	0.549	N.A.	0.543	0.227
90-DH	0.436	N.A.	0.462	0.120	0.616	N.A.	0.573	0.234
60-DH	0.441	N.A.	0.468	0.113	0.633	N.A.	0.579	0.230
30-DH	0.481	N.A.	0.481	0.102	0.596	N.A.	0.604	0.240
14-DH	0.500	N.A.	0.496	0.099	0.632	N.A.	0.606	0.238
7-DH	0.515	N.A.	0.511	0.101	0.603	N.A.	0.610	0.249
3-DH	0.537	N.A.	0.529	0.113	0.619	N.A.	0.608	0.263
1-DH	0.554	N.A.	0.541	0.117	0.603	N.A.	0.597	0.266

Table C-4b: Spectral Regional and At-Site Statistical Characteristics in Real Space–Coefficient of Skewness (Sk)

	Upper Mississippi Basin				Missouri Basin			
Spectral	Regional		At-Site		Regional		At-Site	
Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.697	N.A.	1.179	0.980	0.850	N.A.	1.183	1.031
3-DL	0.653	N.A.	1.176	0.936	0.850	N.A.	1.151	0.963
7-DL	0.630	N.A.	1.160	0.892	0.899	N.A.	1.130	0.907
14-DL	0.659	N.A.	1.156	0.839	0.861	N.A.	1.081	0.929
30-DL	0.741	N.A.	1.243	0.795	0.863	N.A.	1.114	0.972
60-DL	1.225	N.A.	1.951	1.567	0.835	N.A.	1.302	1.341
90-DL	1.286	N.A.	1.776	1.052	1.065	N.A.	1.270	1.056
180-DL	0.916	N.A.	1.032	0.377	1.092	N.A.	1.002	0.585
AM	0.773	N.A.	0.844	0.508	0.942	N.A.	1.026	0.577
180-DH	0.561	N.A.	0.788	0.598	1.021	N.A.	1.034	0.634
90-DH	0.578	N.A.	0.634	0.431	1.418	N.A.	1.109	0.628
60-DH	0.480	N.A.	0.623	0.367	1.397	N.A.	1.156	0.613
30-DH	0.736	N.A.	0.715	0.284	1.264	N.A.	1.356	0.832
14-DH	0.802	N.A.	0.756	0.352	1.768	N.A.	1.354	0.611
7-DH	0.812	N.A.	0.829	0.447	1.441	N.A.	1.366	0.624
3-DH	0.950	N.A.	0.978	0.599	1.496	N.A.	1.340	0.641
1-DH	1.188	N.A.	1.132	0.734	1.593	N.A.	1.378	0.737

Table C-4c: Spectral Regional and At-Site Statistical Characteristics in Real Space–Coefficient of Kurtosis (Ku)

	Upper Mississippi Basin				Missouri Basin			
Spectral	Regional		At-Site		Regional		At-Site	
Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.080	N.A.	2.687	4.805	0.168	N.A.	2.740	4.337
3-DL	-0.091	N.A.	2.531	4.742	0.382	N.A.	2.459	3.851
7-DL	-0.384	N.A.	2.366	4.440	0.684	N.A.	2.243	3.226
14-DL	-0.091	N.A.	2.263	3.940	0.673	N.A.	2.047	2.963
30-DL	0.142	N.A.	2.468	3.817	0.691	N.A.	2.318	2.777
60-DL	1.691	N.A.	7.655	12.177	0.530	N.A.	3.740	5.519
90-DL	1.533	N.A.	5.328	6.649	1.240	N.A.	2.951	4.123
180-DL	1.016	N.A.	1.356	1.448	1.035	N.A.	1.280	1.708
AM	0.859	N.A.	1.609	2.056	0.378	N.A.	1.701	2.302
180-DH	0.329	N.A.	1.633	2.578	1.121	N.A.	1.790	2.613
90-DH	0.554	N.A.	0.805	1.333	2.769	N.A.	1.896	2.301
60-DH	-0.129	N.A.	0.584	1.108	2.514	N.A.	1.987	2.600
30-DH	0.558	N.A.	0.692	0.789	2.115	N.A.	3.020	4.366
14-DH	0.716	N.A.	0.740	1.070	4.888	N.A.	2.733	2.869
7-DH	0.586	N.A.	1.023	1.901	2.946	N.A.	2.874	2.734
3-DH	0.785	N.A.	1.717	3.367	3.139	N.A.	2.870	2.459
1-DH	1.658	N.A.	2.437	4.721	3.901	N.A.	3.387	3.457

Table C-5a: Spectral Regional and At-Site Statistical Characteristics in Log Space– Coefficient of Variation(Cv)

	Upper Mississippi Basin				Missouri Basin			
	Regional		At-Site		Regional		At-Site	
Spectral Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.106	N.A.	0.130	0.079	0.120	N.A.	0.175	0.127
3-DL	0.097	N.A.	0.121	0.079	0.117	N.A.	0.164	0.116
7-DL	0.091	N.A.	0.116	0.075	0.114	N.A.	0.155	0.108
14-DL	0.088	N.A.	0.113	0.072	0.111	N.A.	0.147	0.102
30-DL	0.089	N.A.	0.109	0.066	0.103	N.A.	0.138	0.095
60-DL	0.093	N.A.	0.111	0.063	0.092	N.A.	0.126	0.089
90-DL	0.093	N.A.	0.113	0.063	0.089	N.A.	0.120	0.088
180-DL	0.078	N.A.	0.093	0.039	0.089	N.A.	0.103	0.067
AM	0.067	N.A.	0.071	0.025	0.082	N.A.	0.082	0.041
180-DH	0.065	N.A.	0.068	0.023	0.083	N.A.	0.082	0.039
90-DH	0.065	N.A.	0.067	0.021	0.085	N.A.	0.082	0.040
60-DH	0.064	N.A.	0.066	0.020	0.084	N.A.	0.080	0.040
30-DH	0.065	N.A.	0.065	0.019	0.079	N.A.	0.079	0.039
14-DH	0.064	N.A.	0.064	0.018	0.077	N.A.	0.078	0.038
7-DH	0.064	N.A.	0.063	0.016	0.072	N.A.	0.076	0.038
3-DH	0.062	N.A.	0.062	0.016	0.069	N.A.	0.073	0.037
1-DH	0.060	N.A.	0.061	0.015	0.066	N.A.	0.069	0.036

Table C-5b: Spectral Regional and At-Site Statistical Characteristics in Log Space– Coefficient of Skewness(Sk)

	Upper Mississippi Basin				Missouri Basin			
Spectral	Regional		At-Site		Regional		At-Site	
Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	-0.314	N.A.	-0.388	0.469	-0.164	N.A.	-0.422	0.492
3-DL	-0.244	N.A.	-0.259	0.390	-0.119	N.A.	-0.368	0.429
7-DL	-0.111	N.A.	-0.198	0.351	-0.126	N.A.	-0.342	0.456
14-DL	-0.117	N.A.	-0.164	0.345	-0.174	N.A.	-0.346	0.430
30-DL	-0.063	N.A.	-0.069	0.258	-0.159	N.A.	-0.378	0.433
60-DL	0.081	N.A.	0.151	0.368	-0.014	N.A.	-0.278	0.645
90-DL	0.194	N.A.	0.127	0.309	0.046	N.A.	-0.127	0.376
180-DL	-0.304	N.A.	-0.347	0.352	0.242	N.A.	-0.264	0.501
AM	-0.510	N.A.	-0.576	0.278	-0.185	N.A.	-0.193	0.235
180-DH	-0.692	N.A.	-0.688	0.322	-0.358	N.A.	-0.289	0.243
90-DH	-0.673	N.A.	-0.750	0.322	-0.299	N.A.	-0.321	0.221
60-DH	-0.661	N.A.	-0.741	0.341	-0.286	N.A.	-0.312	0.229
30-DH	-0.592	N.A.	-0.679	0.355	-0.428	N.A.	-0.323	0.258
14-DH	-0.498	N.A.	-0.638	0.382	-0.512	N.A.	-0.359	0.349
7-DH	-0.564	N.A.	-0.610	0.355	-0.450	N.A.	-0.330	0.342
3-DH	-0.458	N.A.	-0.562	0.319	-0.426	N.A.	-0.323	0.384
1-DH	-0.368	N.A.	-0.484	0.319	-0.408	N.A.	-0.346	0.423

Table C-5c: Spectral Regional and At-Site Statistical Characteristics in Log Space– Coefficient of Kurtosis (Ku)

	Upper Mississippi Basin				Missouri Basin			
Spectral	Regional		At-Site		Regional		At-Site	
Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	-0.355	N.A.	0.231	0.973	-0.336	N.A.	0.229	0.683
3-DL	-0.385	N.A.	0.148	0.919	-0.178	N.A.	0.046	0.624
7-DL	-0.737	N.A.	-0.023	0.792	-0.136	N.A.	0.046	0.826
14-DL	-0.731	N.A.	-0.112	0.801	-0.055	N.A.	0.029	0.794
30-DL	-0.582	N.A.	-0.239	0.471	0.129	N.A.	0.258	0.798
60-DL	-0.705	N.A.	-0.185	0.684	0.049	N.A.	0.542	2.040
90-DL	-0.630	N.A.	-0.427	0.406	0.022	N.A.	-0.064	0.375
180-DL	-0.467	N.A.	-0.216	0.443	-0.540	N.A.	-0.085	0.569
AM	0.034	N.A.	0.305	0.730	-0.254	N.A.	-0.368	0.239
180-DH	0.374	N.A.	0.583	0.897	-0.112	N.A.	-0.234	0.293
90-DH	0.190	N.A.	0.647	1.041	-0.150	N.A.	-0.077	0.412
60-DH	0.182	N.A.	0.674	1.129	-0.175	N.A.	-0.010	0.500
30-DH	0.041	N.A.	0.601	1.102	-0.052	N.A.	0.131	0.665
14-DH	-0.120	N.A.	0.552	1.054	0.348	N.A.	0.291	0.829
7-DH	-0.106	N.A.	0.514	1.001	0.089	N.A.	0.205	0.717
3-DH	-0.044	N.A.	0.497	0.922	0.047	N.A.	0.215	0.850
1-DH	0.016	N.A.	0.458	0.830	0.136	N.A.	0.353	1.108

Table C-6a: Spectral Regional and At-Site Statistical Characteristics in Real Space via Log Space– Coefficient of Variation (Cv)

	Upper Mississippi Basin				Missouri Basin			
Spectral	Regional		At-Site		Regional		At-Site	
Element	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.557	0.198	0.576	0.271	0.571	0.186	0.621	0.351
3-DL	0.499	0.172	0.551	0.275	0.562	0.184	0.610	0.347
7-DL	0.480	0.165	0.540	0.271	0.561	0.185	0.596	0.347
14-DL	0.471	0.157	0.538	0.267	0.548	0.177	0.591	0.353
30-DL	0.503	0.165	0.551	0.270	0.529	0.166	0.592	0.369
60-DL	0.545	0.173	0.675	0.373	0.509	0.148	0.644	0.474
90-DL	0.622	0.187	0.693	0.335	0.515	0.139	0.617	0.437
180-DL	0.588	0.099	0.561	0.167	0.591	0.144	0.527	0.288
AM	0.459	0.081	0.461	0.133	0.563	0.108	0.517	0.235
180-DH	0.458	0.072	0.464	0.134	0.581	0.103	0.543	0.227
90-DH	0.467	0.068	0.462	0.120	0.648	0.114	0.573	0.234
60-DH	0.470	0.066	0.468	0.113	0.651	0.111	0.579	0.230
30-DH	0.487	0.072	0.481	0.102	0.606	0.098	0.604	0.240
14-DH	0.523	0.066	0.496	0.099	0.625	0.102	0.606	0.238
7-DH	0.521	0.057	0.511	0.101	0.592	0.094	0.610	0.249
3-DH	0.542	0.057	0.529	0.113	0.582	0.093	0.608	0.263
1-DH	0.553	0.058	0.541	0.117	0.578	0.094	0.597	0.266

Table C-6b: Spectral Regional and At-Site Statistical Characteristics in Real Space via Log Space– Coefficient of Skewness (Sk)

Spectral Element	Upper Mississippi Basin				Missouri Basin			
	Regional		At-Site		Regional		At-Site	
	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.840	0.378	1.179	0.980	1.098	0.322	1.183	1.031
3-DL	0.790	0.336	1.176	0.936	1.169	0.365	1.151	0.963
7-DL	0.766	0.303	1.160	0.892	1.247	0.405	1.130	0.907
14-DL	0.724	0.278	1.156	0.839	1.189	0.380	1.081	0.929
30-DL	0.956	0.326	1.243	0.795	1.258	0.370	1.114	0.972
60-DL	1.246	0.414	1.951	1.567	1.208	0.283	1.302	1.341
90-DL	1.506	0.410	1.776	1.052	1.187	0.236	1.270	1.056
180-DL	1.156	0.280	1.032	0.377	1.432	0.284	1.002	0.585
AM	0.782	0.240	0.844	0.508	1.203	0.267	1.026	0.577
180-DH	0.661	0.215	0.788	0.598	1.263	0.312	1.034	0.634
90-DH	0.562	0.180	0.634	0.431	1.530	0.338	1.109	0.628
60-DH	0.547	0.163	0.623	0.367	1.416	0.289	1.156	0.613
30-DH	0.662	0.189	0.715	0.284	1.294	0.288	1.356	0.832
14-DH	0.883	0.187	0.756	0.352	1.702	0.377	1.354	0.611
7-DH	0.676	0.134	0.829	0.447	1.346	0.292	1.366	0.624
3-DH	0.897	0.137	0.978	0.599	1.250	0.276	1.340	0.641
1-DH	1.091	0.143	1.132	0.734	1.393	0.305	1.378	0.737

Table C-6c: Spectral Regional and At-Site Statistical Characteristics in Real Space via Log Space – Coefficient of Kurtosis (Ku)

Spectral Element	Upper Mississippi Basin				Missouri Basin			
	Regional		At-Site		Regional		At-Site	
	Mean	Std. Dev.	Mean	Std. Dev	Mean	Std. Dev.	Mean	Std. Dev
1-DL	0.350	0.983	2.687	4.805	0.784	0.768	2.740	4.337
3-DL	0.261	0.895	2.531	4.742	1.243	0.951	2.459	3.851
7-DL	0.106	0.805	2.366	4.440	1.749	1.227	2.243	3.226
14-DL	-0.053	0.685	2.263	3.940	1.456	1.013	2.047	2.963
30-DL	0.697	1.005	2.468	3.817	1.706	0.953	2.318	2.777
60-DL	2.247	2.137	7.655	12.177	1.178	0.627	3.740	5.519
90-DL	2.894	2.048	5.328	6.649	0.850	0.437	2.951	4.123
180-DL	1.977	1.150	1.356	1.448	2.177	0.966	1.280	1.708
AM	0.952	0.690	1.609	2.056	1.850	0.853	1.701	2.302
180-DH	0.620	0.481	1.633	2.578	2.764	1.206	1.790	2.613
90-DH	0.149	0.340	0.805	1.333	3.635	1.371	1.896	2.301
60-DH	-0.022	0.264	0.584	1.108	2.721	1.035	1.987	2.600
30-DH	0.340	0.423	0.692	0.789	2.481	0.899	3.020	4.366
14-DH	1.079	0.573	0.740	1.070	5.011	1.536	2.733	2.869
7-DH	0.110	0.259	1.023	1.901	2.756	0.880	2.874	2.734
3-DH	0.698	0.350	1.717	3.367	2.367	0.815	2.870	2.459
1-DH	1.347	0.437	2.437	4.721	3.212	1.023	3.387	3.457

Let ξ denote one of the three statistical descriptors, either the Cv, Sk or Ku., and let j_1 and j_2 index the sites in basin 1, the Upper Mississippi, and in basin 2, the Missouri, where, $j_1 = 1,\ldots, M_1 = 21$ and $j_2 = 1,\ldots, M_2 = 11$. The descriptor for the κ spectral sequence at the $j_i - th$ site, where $i = 1, 2$, is denoted as ξ_{κ, j_i}. The at-site means and standard deviations of ξ_{κ, j_i} are given by

$$u_\kappa(\xi) = \sum_{j_1=1}^{M_i} \xi_{\kappa, j_1} \Big/ M_i \tag{C-1}$$

$$v_\kappa\left(\tilde{\xi}\right) = \left\{\left[\sum_{j_i=1}^{M_i} \tilde{\xi}^2_{\kappa, j_i} \Big/ M_i\right] - u^2_\kappa\left(\tilde{\xi}\right)\right\}^{1/2} \tag{C-2}$$

The 608 sequences combined into 38 regionalized sequences, of which 19 relate to each of the 19 spectral elements for each of the two basins. In real space or log space, for any spectral element, the regionalized sequences at each site in a basin has the same statistical discription as the regional sequence itself. The individual sequences differ only in terms of factors of proportionality, where the factors are the medians of the at-site flow sequences. Thus, for a given basin, the regional descripitors are the “means” of the descriptors of the regionalized sequences, whereby, the standard deviations among the descriptors of the regionalized sequences are equal to zero. Refer to Tables 1a, 1b and 1c and 2a, 2b and 2c, above.

Let ϕ_{κ, j_i} and θ_{κ, j_i} denote one the three statistical descriptors in real space and log space, respectively, of the regionalized κ spectral sequence at the $j_i - th$ site. It follows that $\phi_{\kappa, j_i} = \phi_\kappa \ \forall j_i$ and $\theta_{\kappa, j_i} = \theta_\kappa \ \forall j_i$, whereby

$$u_\kappa(\phi) = \phi_\kappa \tag{C-3}$$

$$v_\kappa(\phi) = 0 \tag{C-4}$$

and

$$u_\kappa(\theta) = \theta_\kappa \tag{C-5}$$

$$v_{\kappa}(\theta) = 0 \tag{C-6}$$

If a regional distribution in log space is transformed into real space, then the descriptors of the regionalized sequences at the various sites will in general differ. Thus, a non-zero valued standard deviation accompanies the mean of the set of regionalized descriptors. The spectral descriptors of the regionalized sequences for the two basins, as well as the regional means and standard deviations of the descriptors are given in Tables C-3a, C-3b and C-3c. Refer to Tables C-5a, C-5b and C-5c above.

Let $\tilde{\xi}_{\kappa, j_i}$ denote one of the three descriptors derived via transformation from log space to real space. Refer to the discussion of the log space to real space transformation, above. The mean and standard deviation of $\tilde{\xi}_{\kappa, j_i}$ over j_i are given by

$$u_{\kappa}\left(\tilde{\xi}\right) = \sum_{j_i=1}^{M_i} \tilde{\xi}_{\kappa, j_i} \Big/ M_i \tag{C-7}$$

$$v_{\kappa}\left(\tilde{\xi}\right) = \left\{ \left[\sum_{j_i=1}^{M_i} \tilde{\xi}^2_{\kappa, j_i} \Big/ M_i \right] - u^2_{\kappa}\left(\tilde{\xi}\right) \right\}^{1/2} \tag{C-8}$$

Refer to Tables C-6a, C-6b and C-6c, above.

Appendix D: De-Markoving and De-trending Flow Sequences

Let $\{x_\tau: \tau = T_1, \ldots, T_n\}$ be an arbitrary observed sequence of annual flows spanning a period of n years from T_1 to T_n. The sequence can be represented as $\{x_t; t = 1, \ldots, n\}$, where $t = T_t - T_0$ and T_0 denotes the year prior to the year of the initial observation, T_1.

Estimate of Trend

The flow at time t is presumed to be the sum of a random component, z_t, and a component, βt:

$$x_t = z_t + \beta t \tag{D-1}$$

where $|\beta| \geq 0$. If $\beta = 0$, then $x_t = z_t$, whereby the sequence $\{x_t; t = 1, \ldots, n\}$ is a realization of a stationary stochastic process. If $\beta \neq 0$, then $\{x_t; t = 1, \ldots, n\}$ is a realization of a stochastic process that is nonstationary in the mean. The rate at which the mean changes with time is a constant equal to β.

The estimate of trend is given by the regression of the time on flow:

$$\hat{x}_t = \bar{x} + b(t - \bar{t}) \tag{D-2}$$

where

$$\bar{x} = \frac{1}{n}\sum_{t=1}^{n} x_t \tag{D-3}$$

$$\begin{aligned} \bar{t} &= \frac{1}{n}\sum_{t=1}^{n} t \\ &= \frac{n+1}{2} \end{aligned} \tag{D-4}$$

$$\begin{aligned} b &= \frac{Cov(x,t)}{s_t^2} \\ &= \frac{\frac{1}{n}\sum_{t=1}^{n} x_t t - \bar{x}\bar{t}}{s_t^2} \end{aligned} \tag{D-5}$$

and

$$s_t^2 = \frac{1}{n}\sum_{t=1}^{n} t^2 - \bar{t}^2 = \frac{n^2-1}{12} \qquad \text{(D-6)}$$

The coefficient b is an estimate of β.

The correlation between x and t is given by

$$R_{x,t} = \frac{Cov(x,t)}{s_x s_t} \qquad \text{(D-7)}$$

where s_t is obtained from Eq. (6) and s_x is obtained from

$$s_x^2 = \frac{1}{n}\sum_{t=1}^{n}\left(x_t - \bar{x}\right)^2 \qquad \text{(D-8)}$$

Let y denote a linear transform of x,

$$x = cy + d \qquad \text{(D-9)}$$

whereby

$$\bar{x} = c\bar{y} + d \qquad \text{(D-10)}$$

$$s_x^2 = c^2 s_y^2 \qquad \text{(D-11)}$$

The estimate of trend for the transformed sequence, $\{y_t : t = 1,\dots,n\}$, is

$$\hat{y} = \bar{y} + b'\left(t - \bar{t}\right) \qquad \text{(D-12)}$$

where

$$\bar{y} = \frac{\bar{x} - d}{c} \qquad \text{(D-13)}$$

$$
\begin{aligned}
b' &= \frac{Cov(y_t, t)}{s_t^2} \\
&= \frac{\frac{1}{n}\sum_{t=1}^{n} y_t t - \bar{y}\bar{t}}{s_t^2} \\
&= \frac{\frac{1}{c}\left[\frac{1}{n}\sum_{t=1}^{n} x_t t - \bar{x}\bar{t}\right]}{s_t^2} \\
&= \frac{b}{c}
\end{aligned}
\qquad \text{(D-14)}
$$

In the case where $d = \bar{x}$ and $c = s_x$, the sequence $\{y_t : t = 1, \ldots, n\}$ is referred to as the sequence of standardized flows for which the estimate of trend is

$$
\begin{aligned}
\hat{y} &= b'(t - \bar{t}) \\
&= \frac{b}{s_x}(t - \bar{t})
\end{aligned}
\qquad \text{(D-15)}
$$

Estimate of Persistence

Markovian persistence is defined as

$$x_{t+1} = \mu + \rho(x_t - \mu) + (1 - \rho^2)^{1/2}\, \sigma\varepsilon_t \qquad \text{(D-16)}$$

where μ and σ denote the mean and standard deviation of x_t $\forall t$, and ε_{t+1} is a random component distributed independently of x_t with zero mean and unit variance $\forall t$. Persistence is measured by the first order autocorrelation coefficient $\rho \equiv \rho_1$, where $-1 \leq \rho \leq 1$. Higher order autocorrelations, ρ_κ, are related to ρ_1 as

$$\rho_\kappa = \rho_1^\kappa \qquad \text{(D-17)}$$

where $|\kappa| = 0, 1, 2, \ldots, \infty$.

If $\rho = 0$, then

$$x_{t+1} = \mu + \sigma\varepsilon_{t+1} \qquad \text{(D-18)}$$

in which case the iid assumption holds.

Under the assumption that $\{x_t; t = 1, \dots, n\}$ is a realization of a stationary stochastic process, the extent to which the process is persistent may be measured by the $k-th$ order autocorrelation coefficient,

$$r_k = \frac{\frac{1}{(n-k)}\sum_{t=1}^{n-k}\left(x_t - \frac{1}{(n-k)}\sum_{t=1}^{n-k}x_t\right)\left(x_{t+k} - \frac{1}{(n-k)}\sum_{t=1}^{n-k}x_{t+k}\right)}{\left\{\left[\frac{1}{(n-k)}\sum_{t=1}^{n-k}\left(x_t - \frac{1}{n-k}\sum_{t=1}^{n-k}x_t\right)\right]\left[\frac{1}{(n-k)}\sum_{t=1}^{n-k}\left(x_{t+k} - \frac{1}{(n-k)}\sum_{t=1}^{n-k}x_{t+k}\right)\right]\right\}^{1/2}} \qquad \text{(D-19)}$$

Residual Trend and Persistence

The coefficients b and r are estimates of the degrees of trend and persistence in a given sequence, $\{x_t: t = 1, \dots, n\}$. To measure the extent to which trend is compromised by persistence and conversely, the sequence may be de-Markoved and de-trended.

De-Markoving yields the sequence $\{u_t: t = 2, \dots, n\}$, where

$$u_{t+1} = \frac{(x_{t+1} - \bar{x}) - r(x_t - \bar{x})}{(1 - r^2)^{1/2} s} \qquad \text{(D-20)}$$

where $\bar{x}$ and s denote the estimates of μ and σ and r denotes the estimate of ρ. Refer to Eqs. (D-3), (D-10) and (D-18). The measure of trend in the de-Markoved sequence, referred to as the residual trend, is denoted as b^*.

De-trending yields the sequence $\{v_t: t = 1, \dots, n\}$ where

$$v_t = x_t - \bar{x} - b(t - \bar{t}) \qquad \text{(D-21)}$$

The measure of persistence in the de-trended sequence, referred to as the residual persistence, is denoted as r^*.

Hypothetical Sequences

The effects of de-Markoving and De-trending on estimates of trend and persistence is illustrated by hypothetical sequences of length $n = 70$ corresponding to the length of the selected sequences in the Upper Mississippi and Missouri basins. Trend is taken to be $\beta = 0(0.002)0.020$ and persistence is taken to be $\rho = 0(0.2)0.8$. It is assumed that the hypothetical flows are normally distributed with zero mean and unit variance. See Table D-1.

Table D-1: Trend and Persistence in Hypothetical Sequences

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			ρ=0			
β=0	-0.002	0.000	0.000	-0.053	-0.052	-0.003
β=0.002	0.006	0.000	0.006	-0.047	-0.061	-0.001
β=0.004	0.008	0.000	0.008	-0.037	-0.061	-0.001
β=0.006	0.010	0.000	0.010	-0.024	-0.061	-0.002
β=0.008	0.012	0.000	0.011	-0.008	-0.061	-0.004
β=0.010	0.013	0.000	0.013	0.009	-0.061	-0.006
β=0.012	0.015	0.000	0.014	0.029	-0.061	-0.009
β=0.014	0.017	0.000	0.015	0.050	-0.061	-0.013
β=0.016	0.018	0.000	0.017	0.072	-0.061	-0.017
β′0.018	0.020	0.000	0.018	0.096	-0.061	-0.023
β=0.020	0.021	0.000	0.018	0.121	-0.061	-0.030
			ρ=0.2			
β=0	-0.002	0.000	0.000	0.142	0.142	0.010
β=0.002	-0.001	0.000	0.001	0.141	0.142	0.010
β=0.004	0.003	0.000	0.004	0.148	0.142	0.011
β=0.006	0.005	0.000	0.006	0.157	0.142	0.011
β=0.008	0.008	0.000	0.008	0.169	0.142	0.010
β=0.010	0.010	0.000	0.009	0.183	0.142	0.008
β=0.012	0.012	0.000	0.011	0.200	0.142	0.005
β′0.014	0.014	0.000	0.012	0.219	0.142	0.002
β=0.016	0.016	0.000	0.014	0.240	0.142	-0.003
β′0.018	0.018	0.000	0.015	0.263	0.142	-0.009
β=0.020	0.019	0.000	0.016	0.286	0.142	-0.015
			ρ=0.4			
β=0	-0.001	0.000	0.000	0.343	0.341	0.017
β=0.002	0.001	0.000	0.002	0.342	0.341	0.018
β=0.004	0.004	0.000	0.004	0.346	0.341	0.017
β=0.006	0.006	0.000	0.005	0.353	0.341	0.015
β′0.008	0.008	0.000	0.006	0.363	0.341	0.012
β=0.010	0.010	0.000	0.008	0.374	0.341	0.009
β=0.012	0.012	0.000	0.009	0.388	0.341	0.005
β=0.014	0.014	0.000	0.010	0.403	0.341	0.000
β=0.016	0.016	0.000	0.011	0.419	0.341	-0.006
β=0.018	0.018	0.000	0.012	0.437	0.341	-0.012
β=0.020	0.020	0.000	0.013	0.455	0.341	-0.019

Table D-1: Assessment of Trend and Persistence in Hypothetical Sequences (Continued)

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
$\rho=0.6$						
$\beta=0$	-0.002	0.000	-0.001	0.548	0.548	0.008
$\beta=0.002$	0.001	0.000	0.001	0.548	0.548	0.010
$\beta=0.004$	0.004	0.000	0.002	0.551	0.548	0.008
$\beta=0.006$	0.006	0.000	0.004	0.556	0.548	0.006
$\beta=0.008$	0.008	0.000	0.005	0.562	0.548	0.003
$\beta=0.010$	0.011	0.000	0.006	0.570	0.548	-0.001
$\beta=0.012$	0.013	0.000	0.007	0.579	0.548	-0.006
$\beta=0.014$	0.015	0.000	0.008	0.590	0.548	-0.011
$\beta=0.016$	0.017	0.000	0.009	0.601	0.548	-0.016
$\beta=0.018$	0.019	0.000	0.009	0.613	0.548	-0.022
$\beta=0.020$	0.020	0.000	0.010	0.626	0.548	-0.028
$\rho=0.8$						
$\beta=0$	-0.005	0.000	-0.003	0.757	0.757	-0.022
$\beta=0.002$	0.000	0.000	-0.001	0.757	0.757	-0.016
$\beta=0.004$	0.002	0.000	0.000	0.757	0.757	-0.017
$\beta=0.006$	0.005	0.000	0.001	0.759	0.757	-0.019
$\beta=0.008$	0.007	0.000	0.002	0.761	0.757	-0.022
$\beta=0.010$	0.010	0.000	0.003	0.765	0.757	-0.025
$\beta=0.012$	0.012	0.000	0.003	0.769	0.757	-0.028
$\beta=0.014$	0.014	0.000	0.004	0.775	0.757	-0.032
$\beta=0.016$	0.016	0.000	0.005	0.781	0.757	-0.036
$\beta=0.018$	0.018	0.000	0.005	0.787	0.757	-0.040
$\beta=0.020$	0.020	0.000	0.006	0.794	0.757	-0.044

From Table D-1 it is noted that de-trending fully accounts for the estimated trend in a sequence, but has little effect on persistence. On the otherhand, de-Markoving almost fully accounts for persistence and more effectively accounts for trend the more persistent the sequence is.

Statistical Significance

To effectively assess the effects of de-trending and de-Markoving, account must be taken of the degree to which the estimates of trend and persistence are statistically significance.

At issue is whether or not the null hypothesis of no trend,

$$H_0 : \beta = 0 \tag{A}$$

holds before de-Markoving and whether or not the null hypothesis

$$H_0 : \beta^* = 0 \tag{B}$$

holds after de-Markoving.

Also at issue is whether or not the null hypothesis

$$H_0 : \rho = 0 \tag{C}$$

holds before de-trending and whether or not the null hypothesis

$$H_0 : \rho^* = 0 \tag{D}$$

after de-trending.

Assume that each of the null hypotheses hold and that x, u and v are each normally distributed. The statistic h, defined as

$$h = R\left[(n-2)/\left(1-R^2\right)\right]^{1/2} \tag{D-22}$$

where h is distributed as Student with $n-2$ degrees of freedom, where $R \equiv R_{x,t}$. Refer to Eq. (D-7). A value of h is significant at the 5% level if $h > h^*$ and h is significant at the 1% level if $h > h^{**}$.

Persistence is taken to be significant at the 5% level or the 1% level if the value of r is greater than $2/\sqrt{n}$ or greater than $3/\sqrt{n}$, respectively.

The effectiveness of de-Markoving and de-trending in reference to sampling errors is illustrated via three 10 sequences from each of three “populations”:

$$Pop.\ 1: \rho = 0, \beta = 0.014$$
$$Pop.\ 2: \rho = 0.4, \beta = 0$$
$$Pop.\ 3: \rho = 0.4, \beta = 0.014$$

For each population, “flows” are normally distributed with zero mean and unit variance. See Table D-2.

Table D-2: Sampling Variability in Trend and Persistence among Hypothetical Sequences

	Trend			Persistence		
Sequence	Obs.	DT	DM	Obs.	DT	DM
$\rho=0, \beta=0.014$						
1	0.007	0.000	0.010	-0.219	-0.248*	0.040
2	0.017**	0.000	0.016**	0.073	-0.050	-0.010
3	0.012*	0.000	0.010	0.027	-0.012	-0.040
4	0.020**	0.000	0.018**	0.197	0.040	0.003
5	0.009	0.000	0.008	0.014	-0.024	-0.001
6	0.017**	0.000	0.017**	0.047	-0.094	-0.004
7	0.021**	0.000	0.015**	0.207	0.071	-0.034
8	0.015**	0.000	0.012*	0.166	0.083	-0.035
9	0.001	0.000	0.000	0.193	0.192	0.021
10	0.020**	0.000	0.018**	0.098	-0.066	-0.015
Average	0.014	0.000	0.012	0.080	-0.011	-0.007
Stdev	0.006	0.000	0.005	0.122	0.113	0.024
$\rho=0.4, \beta=0$						
1	0.005	0.000	0.000	0.507**	0.503**	0.062
2	-0.004	0.000	-0.001	0.170	0.168	-0.020
3	0.002	0.000	0.002	0.299*	0.298*	0.066
4	0.011*	0.000	0.005	0.539**	0.516**	0.004
5	-0.002	0.000	-0.002	0.324*	0.324*	0.082
6	-0.002	0.000	-0.001	0.326*	0.326*	0.012
7	-0.004	0.000	-0.003	0.259*	0.254*	0.009
8	-0.002	0.000	-0.002	0.220	0.219	-0.020
9	0.004	0.000	0.001	0.280*	0.279*	0.130
10	0.011*	0.000	0.007	0.412**	0.385**	0.090
Average	0.002	0.000	0.001	0.334	0.327	0.042
Stdev	0.005	0.000	0.003	0.113	0.107	0.049
$\rho=0.4, \beta'=0.014$						
1	0.017**	0.000	0.011	0.366**	0.280*	0.021
2	0.023**	0.000	0.011*	0.410**	0.240*	-0.046
3	-0.008	0.000	-0.005	0.385**	0.369**	0.094
4	0.027**	0.000	0.015*	0.560**	0.375**	-0.052
5	0.027**	0.000	0.010	0.609**	0.448**	-0.056
6	0.002	0.000	-0.001	0.460**	0.460**	0.085
7	0.024**	0.000	0.011	0.579**	0.439**	-0.074
8	0.011	0.000	0.007	0.354*	0.315*	-0.022
9	0.014**	0.000	0.007	0.390**	0.352*	-0.079
10	0.021**	0.000	0.009	0.541**	0.451**	-0.063
Average	0.016	0.000	0.008	0.465	0.373	-0.019
Stdev	0.011	0.000	0.006	0.092	0.073	0.061

From Table D-2, it is noted that

i) null hypotheses A and C are infrequently rejected when in fact it is true;

2) De-trending fully accounts for trend whether the null hypothesis A is true or not;

3) De-trending yields residual values of persistence that are somewhat smaller than the estimated values of persistence;

4) If the null hypothesis C is true, de-Markoving has little effect on trend but it tends to yield values of residual persistence that are considerably smaller than the estimated values of persistence;

5) If the null hypotheses A and C are false, de-Markoving tends to render significant trend and significant persistence insignificant;

6) If the null hypotheses A and C are false, de-trending fully accounts for trend, but has little effect on the significance of persistence.

The effects of de-trending and de-Markoving the hypothetical flow sequences are shown graphically in Figures D-1 and D-2.

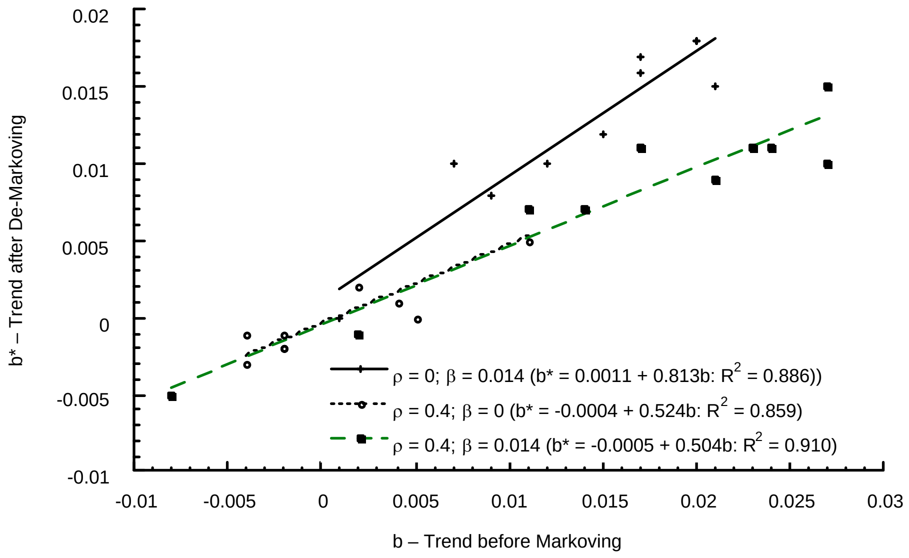


Figure D-1: Effect of De-Markoving on Trend in Hypothetical Sequences

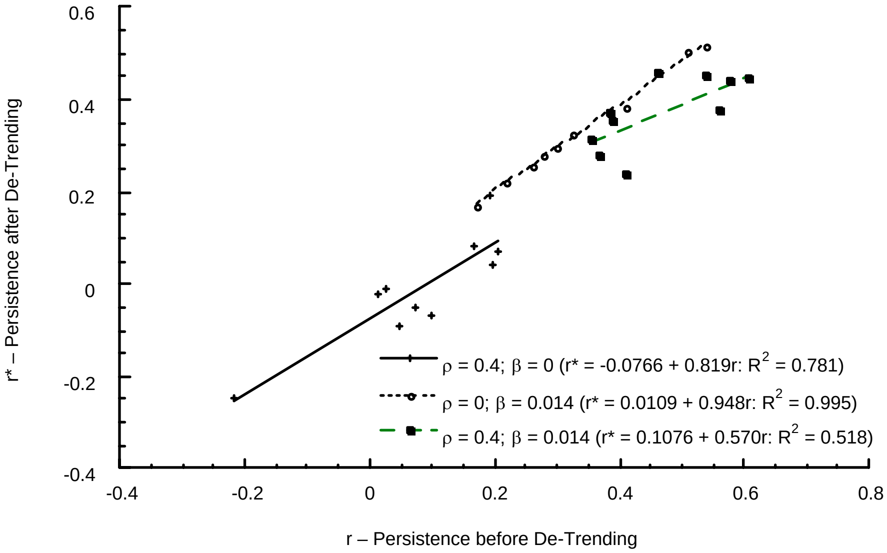


Figure D-2: Effect of De-Trending on Persistence in Hypothetical Sequences

Appendix E: Spectral Trends and Persistence

Assessment of Trends and Persistence

Trends and persistence, as well as residual trends and residual persistence are assessed at the two-tail 5% and 1% levels. See Tables E-1 through E-14.

Table E-1: Trends and Persistence in Annual 3-Day High Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	x
			Upper Mississippi Basin			
St. Croix	0.004	0.000	0.005	0.195	0.189	0.010
Jump	0.000	0.000	0.000	0.027	0.027	-0.002
Black	-0.004	0.000	-0.005	-0.130	-0.139	-0.004
Maquaketa	-0.005	0.000	-0.006	0.084	0.077	-0.002
Mississippi	0.013	0.000	0.011	0.140	0.074	-0.020
Rock	0.005	0.000	0.005	0.027	0.009	0.008
Sugar	-0.011	0.000	-0.011	0.054	0.023	0.019
Pectonica	-0.009	0.000	-0.009	-0.028	-0.049	0.000
Cedar	0.001	0.000	0.001	0.064	0.063	0.011
Skunk	0.009	0.000	0.009	-0.043	-0.077	-0.002
Mississippi	0.016**	0.000	0.013	0.145	0.036	-0.005
Des Moines	0.006	0.000	0.007	-0.034	-0.054	-0.004
Raccoon	0.013**	0.000	0.012*	-0.052	-0.125	0.011
Iroquois	0.017**	0.000	0.013*	0.221	0.119	0.025
Kankakee	0.025**	0.000	0.019**	0.192	-0.108	-0.046
Spoon	0.014**	0.000	0.014*	-0.025	-0.117	-0.001
La Moines	0.017**	0.000	0.014*	0.234	0.125	0.014
Meramec	0.005	0.000	0.003	-0.124	-0.129	0.037
Bourbeuse	0.015**	0.000	0.014*	0.074	-0.014	-0.010
Big	0.015**	0.000	0.013*	0.129	0.042	-0.005
Meramec	0.013**	0.000	0.012*	0.068	-0.002	-0.008
Average	0.008	0.000	0.006	0.058	-0.001	0.001
Stdev	0.009	0.000	0.008	0.105	0.091	0.017
			Missouri Basin			
Yellowstone	0.012**	0.000	0.012	0.139	0.077	0.019
Clarks Fork	0.008	0.000	0.009	0.168	0.143	0.053
Yellowstone	0.011*	0.000	0.010	0.214	0.172	0.055
Big Sioux	0.008	0.000	0.009	-0.040	-0.070	0.008
North Platte	0.002	0.000	0.003	0.002	-0.002	0.010
Bear	-0.003	0.000	-0.004	-0.051	-0.056	-0.013
Elkhorn	0.010	0.000	0.012	-0.121	-0.169	0.004
Nisnabottna	0.014**	0.000	0.011	-0.018	-0.093	0.020
Grand	0.008	0.000	0.008	0.020	-0.013	-0.001
Thompson	0.007	0.000	0.005	0.245*	0.230	-0.002
Gasconade	0.008	0.000	0.008	-0.034	-0.058	0.005
Average	0.006	0.000	0.006	0.069	0.036	0.009
Stdev	0.007	0.000	0.006	0.153	0.155	0.017

Table E-2: Trends and Persistence in Annual 7-Day HighFlows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.005	0.000	0.005	0.183	0.176	0.006
Jump	0.000	0.000	0.000	0.001	0.001	-0.007
Black	-0.003	0.000	-0.003	-0.108	-0.112	-0.004
Maquaketa	0.000	0.000	-0.001	0.070	0.070	-0.001
Mississippi	0.013**	0.000	0.011	0.140	0.078	-0.019
Rock	0.006	0.000	0.005	0.028	0.006	0.008
Sugar	-0.007	0.000	-0.007	0.042	0.033	0.016
Pectonica	-0.006	0.000	-0.006	-0.066	-0.071	-0.013
Cedar	0.004	0.000	0.004	0.035	0.023	0.007
Skunk	0.010*	0.000	0.010	0.001	-0.043	0.001
Mississippi	0.017**	0.000	0.014**	0.144	0.029	-0.007
Des Moines	0.009	0.000	0.009	-0.018	-0.054	-0.001
Raccoon	0.013**	0.000	0.013*	-0.061	-0.140	0.004
Iroquois	0.018**	0.000	0.013**	0.243*	0.129	0.013
Kankakee	0.025**	0.000	0.019**	0.234	-0.050	-0.042
Spoon	0.014**	0.000	0.014**	-0.025	-0.114	-0.003
La Moines	0.015**	0.000	0.013**	0.218	0.130	0.021
Meramec	0.003	0.000	0.001	-0.140	-0.140	0.036
Bourbeuse	0.014**	0.000	0.013**	0.090	0.003	-0.012
Big	0.015**	0.000	0.013**	0.177	0.087	-0.007
Meramec	0.015**	0.000	0.013**	0.101	0.015	-0.011
Average	0.009	0.000	0.007	0.061	0.003	-0.001
Stdev	0.008	0.000	0.007	0.110	0.088	0.015
			Missouri Basin			
Yellowstone	0.012*	0.000	0.012	0.120	0.066	0.015
Clarks Fork	0.007	0.000	0.008	0.113	0.092	0.038
Yellowstone	0.011*	0.000	0.010	0.198	0.155	0.047
Big Sioux	0.009	0.000	0.010	-0.059	-0.095	0.008
North Platte	0.003	0.000	0.003	0.071	0.066	0.021
Bear	-0.002	0.000	-0.003	-0.062	-0.064	-0.017
Elkhorn	0.012**	0.000	0.013	-0.103	-0.170	0.008
Nisnabottna	0.015**	0.000	0.013**	-0.093	-0.185	0.033
Grand	0.008	0.000	0.008**	0.016	-0.016	-0.002
Thompson	0.010	0.000	0.008	0.227	0.195	-0.009
Gasconade	0.008	0.000	0.008	-0.082	-0.107	0.014
Average	0.008	0.000	0.008	0.032	-0.006	0.014
Stdev	0.004	0.000	0.004	0.115	0.123	0.019

Table E-3: Trends and Persistence in Annual 14-Day High Flow

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.005	0.000	0.005	0.200	0.192	-0.003
Jump	-0.002	0.000	-0.002	0.022	0.020	-0.004
Black	-0.001	0.000	0.000	-0.063	-0.063	0.004
Maquaketa	0.005	0.000	0.004	0.061	0.050	-0.003
Mississippi	0.013**	0.000	0.010	0.146	0.086	-0.018
Rock	0.007	0.000	0.006	0.032	0.004	0.008
Sugar	-0.005	0.000	-0.005	0.069	0.064	0.017
Pectonica	-0.001	0.000	-0.001	-0.048	-0.047	-0.014
Cedar	0.010	0.000	0.009	0.029	-0.015	0.003
Skunk	0.012**	0.000	0.011	0.061	-0.002	0.007
Mississippi	0.017**	0.000	0.014*	0.160	0.035	-0.010
Des Moines	0.012*	0.000	0.011	0.011	-0.050	-0.001
Raccoon	0.014**	0.000	0.014*	-0.066	-0.160	0.002
Iroquois	0.019**	0.000	0.013*	0.305*	0.192	0.013
Kankakee	0.026**	0.000	0.018**	0.309*	0.035	-0.037
Spoon	0.016**	0.000	0.015*	0.058	-0.056	-0.011
La Moines	0.015**	0.000	0.012**	0.242*	0.158	0.006
Meramec	0.003	0.000	0.001	-0.071	-0.074	0.022
Bourbeuse	0.016**	0.000	0.014*	0.132	0.025	-0.023
Big	0.014**	0.000	0.012	0.140	0.057	-0.007
Meramec	0.014	0.000	0.012	0.135	0.049	-0.018
Average	0.010	0.000	0.005	0.056	0.024	-0.003
Stdev	0.008	0.000	0.006	0.082	0.085	0.014
			Missouri Basin			
Yellowstone	0.012**	0.000	0.012*	0.076	0.013	0.011
Clarks Fork	0.008	0.000	0.009	0.062	0.036	0.034
Yellowstone	0.012*	0.000	0.011	0.158	0.105	0.033
Big Sioux	0.011	0.000	0.012	-0.021	-0.083	0.008
North Platte	0.003	0.000	0.004	0.092	0.086	0.027
Bear	-0.001	0.000	-0.002	-0.091	-0.092	-0.023
Elkhorn	0.015**	0.000	0.016**	-0.079	-0.178	0.013
Nisnabottna	0.014**	0.000	0.012*	-0.097	-0.173	0.032
Grand	0.010*	0.000	0.011	-0.031	-0.086	0.005
Thompson	0.013*	0.000	0.000	0.000	0.000	0.000
Gasconade	0.007	0.000	0.007	-0.089	-0.113	0.013
Average	0.009	0.000	0.007	-0.002	-0.044	0.014
Stdev	0.005	0.000	0.005	0.083	0.093	0.017

Table E-4: Trends and Persistence in Annual 30-Day High Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.004	0.000	0.004	0.191	0.185	-0.008
Jump	-0.005	0.000	-0.005	0.033	0.026	0.004
Black	-0.002	0.000	-0.001	0.040	0.040	0.017
Maquaketa	0.009	0.000	0.006	0.151	0.121	-0.014
Mississippi	0.013**	0.000	0.010	0.157	0.092	-0.016
Rock	0.008	0.000	0.007	0.064	0.025	0.014
Sugar	-0.001	0.000	-0.002	0.054	0.055	0.006
Pectonica	0.002	0.000	0.002	0.035	0.030	0.000
Cedar	0.014**	0.000	0.011	0.109	0.030	0.006
Skunk	0.014**	0.000	0.012*	0.081	0.003	0.012
Mississippi	0.018**	0.000	0.015*	0.160	0.021	-0.013
Des Moines	0.016**	0.000	0.015*	0.067	-0.037	-0.013
Raccoon	0.016**	0.000	0.015*	-0.027	-0.135	0.006
Iroquois	0.020**	0.000	0.015*	0.243*	0.099	0.005
Kankakee	0.023**	0.000	0.017**	0.286*	0.071	-0.012
Spoon	0.018**	0.000	0.016**	0.117	-0.021	-0.018
La Moines	0.016**	0.000	0.012	0.305*	0.212	0.011
Meramec	0.008	0.000	0.006	0.062	0.041	0.004
Bourbeuse	0.015**	0.000	0.011	0.249*	0.168	-0.020
Big	0.015**	0.000	0.012	0.216	0.128	-0.005
Meramec	0.016**	0.000	0.012*	0.236	0.144	-0.016
Average	0.011	0.000	0.009	0.135	0.062	-0.002
Stdev	0.008	0.000	0.006	0.092	0.079	0.012
			Missouri Basin			
Yellowstone	0.013**	0.000	0.013*	0.056	-0.016	0.007
Clarks Fork	0.007	0.000	0.008	-0.039	-0.061	0.027
Yellowstone	0.012*	0.000	0.011	0.147	0.092	0.019
Big Sioux	0.015**	0.000	0.015*	0.071	-0.029	-0.003
North Platte	0.006	0.000	0.006	0.116	0.098	0.028
Bear	0.000	0.000	-0.001	-0.096	-0.095	-0.025
Elkhorn	0.017**	0.000	0.017**	-0.013	-0.145	0.002
Nisnabottna	0.016**	0.000	0.014*	-0.049	-0.154	0.016
Grand	0.012**	0.000	0.013*	-0.043	-0.115	0.008
Thompson	0.013**	0.000	0.012*	0.049	-0.022	-0.011
Gasconade	0.009	0.000	0.008	-0.018	-0.051	0.000
Average	0.011	0.000	0.011	0.016	-0.045	0.006
Stdev	0.005	0.000	0.005	0.072	0.080	0.015

Table E-5: Trends and Persistence in Annual 60-Day High Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
Upper Mississippi Basin						
St. Croix	0.008	0.000	0.007	0.223	0.202	0.008
Jump	-0.005	0.000	-0.004	0.117	0.112	0.015
Black	0.003	0.000	0.003	0.060	0.054	0.012
Maquaketa	0.011*	0.000	0.008	0.182	0.138	-0.017
Mississippi	0.015**	0.000	0.012	0.240*	0.149	0.004
Rock	0.010*	0.000	0.008	0.156	0.107	0.033
Sugar	0.003	0.000	0.002	0.112	0.105	0.013
Pectonica	0.007	0.000	0.006	0.088	0.063	0.007
Cedar	0.017**	0.000	0.014*	0.169	0.053	-0.018
Skunk	0.013**	0.000	0.012	0.084	0.005	0.013
Mississippi	0.019**	0.000	0.014*	0.223	0.082	-0.001
Des Moines	0.019**	0.000	0.016**	0.173	0.028	-0.042
Raccoon	0.019**	0.000	0.015**	0.114	-0.029	-0.008
Iroquois	0.017**	0.000	0.012*	0.229	0.128	0.008
Kankakee	0.021**	0.000	0.015*	0.232	0.057	-0.023
Spoon	0.017**	0.000	0.015*	0.063	-0.070	-0.012
La Moines	0.015**	0.000	0.011	0.237	0.151	0.016
Meramec	0.006	0.000	0.005	0.053	0.037	0.006
Bourbeuse	0.011*	0.000	0.009	0.121	0.068	-0.007
Big	0.010*	0.000	0.009	0.103	0.061	0.005
Meramec	0.013**	0.000	0.010	0.163	0.096	-0.005
Average	0.012	0.000	0.009	0.150	0.076	0.000
Stdev	0.007	0.000	0.005	0.062	0.062	0.016
Missouri Basin						
Yellowstone	0.014**	0.000	0.013*	0.124	0.050	-0.012
Clarks Fork	0.008	0.000	0.009	0.062	0.034	0.026
Yellowstone	0.013**	0.000	0.012	0.207	0.144	0.012
Big Sioux	0.019**	0.000	0.016**	0.220	0.079	-0.034
North Platte	0.006	0.000	0.006	0.129	0.108	0.032
Bear	0.001	0.000	0.000	-0.057	-0.057	-0.015
Elkhorn	0.019**	0.000	0.016**	0.127	-0.007	-0.037
Nisnabottna	0.020**	0.000	0.016**	0.046	-0.118	-0.012
Grand	0.013**	0.000	0.014*	-0.022	-0.107	0.003
Thompson	0.013**	0.000	0.012*	0.000	-0.073	0.000
Gasconade	0.008	0.000	0.007	0.022	-0.005	-0.004
Average	0.012	0.000	0.011	0.078	0.004	-0.004
Stdev	0.006	0.000	0.005	0.087	0.083	0.021

Table E-6: Trends and Persistence in Annual 90-Day High Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.008	0.000	0.007	0.322*	0.301*	-0.004
Jump	-0.006	0.000	-0.005	0.170	0.161	0.001
Black	0.004	0.000	0.004	0.050	0.042	0.007
Maquaketa	0.013**	0.000	0.008	0.233	0.180	-0.020
Mississippi	0.017**	0.000	0.013*	0.276*	0.166	-0.002
Rock	0.017*	0.000	0.008	0.198	0.142	0.033
Sugar	0.006	0.000	0.005	0.124	0.106	0.004
Pectonica	0.009	0.000	0.007	0.131	0.093	0.004
Cedar	0.019**	0.000	0.014*	0.225	0.087	-0.033
Skunk	0.013**	0.000	0.011	0.108	0.034	0.008
Mississippi	0.020**	0.000	0.014*	0.284*	0.134	-0.015
Des Moines	0.020**	0.000	0.016**	0.230	0.075	-0.061
Raccoon	0.020**	0.000	0.015**	0.169	0.016	-0.022
Iroquois	0.015**	0.000	0.011	0.190	0.107	0.002
Kankakee	0.021**	0.000	0.015*	0.262*	0.085	-0.024
Spoon	0.016**	0.000	0.015*	0.044	-0.083	-0.008
La Moines	0.015**	0.000	0.011	0.195	0.106	0.004
Meramec	0.007	0.000	0.005	0.035	0.016	0.004
Bourbeuse	0.010*	0.000	0.009	0.029	-0.017	-0.001
Big	0.009	0.000	0.008	0.062	0.028	0.005
Meramec	0.011*	0.000	0.010	0.082	0.026	-0.001
Average	0.013	0.000	0.010	0.163	0.086	-0.006
Stdev	0.007	0.000	0.005	0.090	0.082	0.019
			Missouri Basin			
Yellowstone	0.014**	0.000	0.013*	0.157	0.081	-0.037
Clarks Fork	0.008	0.000	0.009	0.087	0.058	0.014
Yellowstone	0.014**	0.000	0.012*	0.228	0.157	-0.004
Big Sioux	0.020**	0.000	0.015*	0.287*	0.146	-0.048
North Platte	0.005	0.000	0.005	0.117	0.102	0.026
Bear	0.001	0.000	0.000	-0.068	-0.067	-0.017
Elkhorn	0.019**	0.000	0.015*	0.199	0.066	-0.058
Nisnabottna	0.022**	0.000	0.017**	0.098	-0.097	-0.032
Grand	0.013**	0.000	0.013*	0.018	-0.068	-0.008
Thompson	0.014**	0.000	0.013*	0.039	-0.043	-0.012
Gasconade	0.009	0.000	0.008	0.034	0.002	-0.003
Average	0.013	0.000	0.011	0.109	0.031	-0.016
Stdev	0.06	0.000	0.005	0.103	0.090	0.026

Table E-7: Trends and Persistence in Annual 180-Day High Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.011*	0.000	0.008	0.467**	0.436**	0.007
Jump	-0.001	0.000	0.000	0.212	0.213	0.031
Black	0.006	0.000	0.006	0.116	0.099	0.015
Maquaketa	0.013**	0.000	0.009	0.288*	0.231	-0.011
Mississippi	0.019**	0.000	0.013*	0.364**	0.244*	-0.010
Rock	0.017**	0.000	0.011	0.301*	0.198	0.035
Sugar	0.018*	0.000	0.008	0.226	0.176	0.010
Pectonica	0.012*	0.000	0.008	0.216	0.159	0.020
Cedar	0.020**	0.000	0.015*	0.244*	0.099	-0.030
Skunk	0.014**	0.000	0.011	0.139	0.061	0.010
Mississippi	0.022**	0.000	0.015*	0.333*	0.164	-0.027
Des Moines	0.020**	0.000	0.015*	0.257*	0.117	-0.054
Raccoon	0.021**	0.000	0.015**	0.228	0.070	-0.021
Iroquois	0.017**	0.000	0.011	0.258*	0.154	-0.005
Kankakee	0.024**	0.000	0.016**	0.319*	0.092	-0.049
Spoon	0.015**	0.000	0.012*	0.059	-0.043	0.003
La Moines	0.014**	0.000	0.009	0.191	0.119	0.010
Meramec	0.010	0.000	0.007	0.162	0.131	0.006
Bourbeuse	0.012**	0.000	0.010	0.147	0.089	-0.015
Big	0.011*	0.000	0.009	0.190	0.150	0.020
Meramec	0.014**	0.000	0.010	0.219	0.155	0.001
Average	0.015	0.000	0.010	0.235	0.148	-0.002
Stdev	0.006	0.000	0.004	0.089	0.090	0.023
			Missouri Basin			
Yellowstone	0.015**	0.000	0.013*	0.142	0.060	-0.041
Clarks Fork	0.008	0.000	0.008	0.070	0.046	0.008
Yellowstone	0.014**	0.000	0.012*	0.238	0.167	-0.017
Big Sioux	0.020**	0.000	0.017*	0.368**	0.235	-0.053
North Platte	0.007	0.000	0.006	0.118	0.095	0.014
Bear	0.002	0.000	0.001	-0.048	-0.049	-0.017
Elkhorn	0.022**	0.000	0.014*	0.350*	0.207	-0.114
Nisnabottna	0.023**	0.000	0.016**	0.192	-0.007	-0.051
Grand	0.014**	0.000	0.013*	0.055	-0.033	-0.011
Thompson	0.014**	0.000	0.011	0.107	0.029	-0.014
Gasconade	0.011*	0.000	0.009	0.203	0.158	-0.012
Average	0.014	0.000	0.011	0.163	0.083	-0.028
Stdev	0.06	0.000	0.005	0.119	0.093	0.034

Table E-8: Trends and Persistence in Annual 180-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.021**	0.000	0.014*	0.395**	0.248*	-0.021
Jump	0.008	0.000	0.008	0.059	0.025	0.019
Black	0.010	0.000	0.010	0.037	-0.011	0.008
Maquaketa	0.013**	0.000	0.009	0.376**	0.321*	0.034
Mississippi	0.020**	0.000	0.013*	0.372**	0.227	-0.033
Rock	0.017**	0.000	0.011	0.348*	0.232	0.029
Sugar	0.020**	0.000	0.012*	0.441**	0.313*	0.022
Pectonica	0.016**	0.000	0.011	0.379**	0.289*	0.047
Cedar	0.016**	0.000	0.010	0.384**	0.304*	0.029
Skunk	0.011*	0.000	0.007	0.281*	0.243*	0.080
Mississippi	0.021**	0.000	0.013*	0.401**	0.252*	-0.013
Des Moines	0.017**	0.000	0.013*	0.319*	0.218	0.036
Raccoon	0.018**	0.000	0.014*	0.285*	0.171	0.070
Iroquois	0.014**	0.000	0.010	0.192	0.119	0.019
Kankakee	0.023**	0.000	0.015*	0.320*	0.124	-0.060
Spoon	0.010*	0.000	0.007	0.102	0.056	0.013
La Moines	0.007	0.000	0.005	0.134	0.108	0.024
Meramec	0.012*	0.000	0.008	0.296*	0.259*	-0.006
Bourbeuse	0.007	0.000	0.004	0.170	0.158	-0.025
Big	0.008	0.000	0.006	0.306*	0.288*	0.039
Meramec	0.012**	0.000	0.008	0.297*	0.256*	0.000
Average	0.014	0.000	0.010	0.281	0.200	0.015
Stdev	0.005	0.000	0.003	0.116	0.095	0.033
			Missouri Basin			
Yellowstone	0.017**	0.000	0.010	0.296*	0.211	-0.088
Clarks Fork	0.011*	0.000	0.007	0.326*	0.300*	-0.032
Yellowstone	0.015**	0.000	0.008	0.402**	0.336*	-0.004
Big Sioux	0.022**	0.000	0.012	0.527**	0.409**	-0.083
North Platte	0.016**	0.000	0.007	0.423**	0.364**	-0.164
Bear	0.012*	0.000	0.008	0.218	0.177	-0.037
Elkhorn	0.026**	0.000	0.014*	0.572**	0.403**	-0.033
Nisnabottna	0.017**	0.000	0.014*	0.182	0.060	0.011
Grand	0.008	0.000	0.005	0.175	0.141	0.029
Thompson	0.009	0.000	0.006	0.198	0.165	0.035
Gasconade	0.007	0.000	0.004	0.353**	0.340*	0.012
Average	0.015	0.000	0.009	0.334	0.264	-0.032
Stdev	0.006	0.000	0.003	0.131	0.113	0.057

Table E-9 : Trends and Persistence in Annual 90-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.026**	0.000	0.014*	0.576**	0.414**	0.000
Jump	0.018**	0.000	0.016**	0.202	0.076	0.040
Black	0.019**	0.000	0.014*	0.379**	0.262*	-0.012
Maquaketa	0.015**	0.000	0.009	0.447**	0.384**	0.056
Mississippi	0.023**	0.000	0.012*	0.494**	0.355*	-0.076
Rock	0.025**	0.000	0.016**	0.358*	0.113	-0.043
Sugar	0.025**	0.000	0.014*	0.505**	0.336*	-0.024
Pectonica	0.022**	0.000	0.013*	0.489**	0.349*	0.016
Cedar	0.020**	0.000	0.013*	0.400**	0.281*	0.031
Skunk	0.013**	0.000	0.009	0.257*	0.196	0.023
Mississippi	0.025**	0.000	0.014*	0.509**	0.334*	-0.013
Des Moines	0.019**	0.000	0.014*	0.338*	0.214	-0.015
Raccoon	0.016**	0.000	0.014*	0.202	0.098	0.026
Iroquois	0.016**	0.000	0.015*	0.160	0.051	0.007
Kankakee	0.024**	0.000	0.014*	0.395**	0.210	-0.071
Spoon	0.009	0.000	0.007	0.062	0.023	0.010
La Moines	0.003	0.000	0.004	-0.005	-0.015	-0.004
Meramec	0.010*	0.000	0.009	0.101	0.062	0.002
Bourbeuse	0.007	0.000	0.007	-0.044	-0.062	-0.002
Big	0.004	0.000	0.003	-0.028	-0.034	-0.006
Meramec	0.006	0.000	0.006	-0.015	-0.030	-0.004
Average	0.016	0.000	0.011	0.275	0.172	-0.003
Stdev	0.007	0.000	0.004	0.199	0.152	0.032
			Missouri Basin			
Yellowstone	0.016**	0.000	0.009	0.307*	0.232	-0.143
Clarks Fork	0.015**	0.000	0.008	0.498**	0.457**	-0.030
Yellowstone	0.016**	0.000	0.009	0.425**	0.353*	-0.054
Big Sioux	0.026**	0.000	0.014*	0.540**	0.362**	-0.192
North Platte	0.023**	0.000	0.009	0.547**	0.436**	-0.135
Bear	0.022**	0.000	0.014*	0.380**	0.230	-0.071
Elkhorn	0.028**	0.000	0.008	0.750**	0.637**	-0.102
Nisnabottna	0.019**	0.000	0.014*	0.241*	0.102	-0.011
Grand	0.009	0.000	0.006	0.105	0.073	0.029
Thompson	0.011*	0.000	0.008	0.106	0.061	0.022
Gasconade	0.006	0.000	0.006	0.042	0.024	0.000
Average	0.017	0.000	0.009	0.358	0.270	-0.063
Stdev	0.007	0.000	0.003	0.211	0.187	0.070

Table E-10: Trends and Persistence in Annual 60-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.025**	0.000	0.013*	0.620**	0.490**	0.046
Jump	0.023**	0.000	0.018**	0.353*	0.167	0.070
Black	0.024**	0.000	0.014*	0.542**	0.393**	0.085
Maquaketa	0.016**	0.000	0.009	0.476**	0.409**	0.053
Mississippi	0.023**	0.000	0.011	0.530**	0.404**	-0.081
Rock	0.026**	0.000	0.015*	0.420**	0.167	-0.016
Sugar	0.029**	0.000	0.013*	0.623**	0.430**	0.056
Pectonica	0.024**	0.000	0.012*	0.540**	0.389**	0.044
Cedar	0.021**	0.000	0.012*	0.405**	0.273*	0.022
Skunk	0.011*	0.000	0.007	0.161	0.121	0.016
Mississippi	0.025**	0.000	0.014*	0.488**	0.305*	-0.008
Des Moines	0.020**	0.000	0.014*	0.416**	0.294*	-0.037
Raccoon	0.016**	0.000	0.014*	0.117	0.017	0.005
Iroquois	0.014**	0.000	0.013*	0.116	0.035	-0.001
Kankakee	0.024**	0.000	0.014*	0.439**	0.265*	-0.123
Spoon	0.011*	0.000	0.010	0.059	0.007	0.004
La Moines	0.007	0.000	0.008	0.036	0.011	-0.003
Meramec	0.014**	0.000	0.011	0.248*	0.181	-0.005
Bourbeuse	0.014**	0.000	0.013*	0.063	-0.018	-0.013
Big	0.009	0.000	0.008	0.087	0.054	-0.003
Meramec	0.012**	0.000	0.010	0.122	0.066	-0.003
Average	0.018	0.000	0.012	0.327	0.212	0.005
Stdev	0.006	0.000	0.003	0.200	0.162	0.047
			Missouri Basin			
Yellowstone	0.016	0.000	0.009	0.292*	0.218	-0.144
Clarks Fork	0.008	0.000	0.004	0.399**	0.388**	0.032
Yellowstone	0.017**	0.000	0.009	0.439**	0.358*	-0.068
Big Sioux	0.027**	0.000	0.012*	0.623**	0.467**	-0.198
North Platte	0.022**	0.000	0.008	0.589**	0.497**	-0.079
Bear	0.023**	0.000	0.013*	0.437**	0.286*	-0.060
Elkhorn	0.028**	0.000	0.006	0.772**	0.671**	-0.080
Nisnabottna	0.019**	0.000	0.014*	0.241*	0.105	-0.018
Grand	0.011*	0.000	0.009	0.077	0.034	0.012
Thompson	0.010*	0.000	0.009	0.075	0.038	0.006
Gasconade	0.010*	0.000	0.008	0.273*	0.239	-0.018
Average	0.017	0.000	0.009	0.383	0.300	-0.056
Stdev	0.007	0.000	0.003	0.210	0.192	0.066

Table E-11: Trends and Persistence in Annual 30-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
Upper Mississippi Basin						
St. Croix	0.024**	0.000	0.012*	0.671**	0.564**	0.076
Jump	0.023**	0.000	0.019**	0.305*	0.092	0.023
Black	0.026**	0.000	0.019**	0.362**	0.113	0.010
Maquaketa	0.016**	0.000	0.008	0.499**	0.435**	0.068
Mississippi	0.021**	0.000	0.010	0.511**	0.401**	-0.042
Rock	0.025**	0.000	0.014*	0.446**	0.228	-0.002
Sugar	0.031**	0.000	0.011	0.697**	0.501**	0.080
Pectonica	0.025**	0.000	0.011	0.578**	0.423**	0.045
Cedar	0.021**	0.000	0.012	0.426**	0.294*	0.045
Skunk	0.017**	0.000	0.009	0.317*	0.232	0.049
Mississippi	0.027**	0.000	0.013*	0.542**	0.352*	-0.005
Des Moines	0.021**	0.000	0.014*	0.448**	0.312*	-0.023
Raccoon	0.016**	0.000	0.015*	0.108	-0.001	0.002
Iroquois	0.016**	0.000	0.013*	0.246*	0.164	0.023
Kankakee	0.021**	0.000	0.014*	0.301*	0.146	-0.069
Spoon	0.013**	0.000	0.010	0.186	0.122	0.038
La Moines	0.005	0.000	0.005	0.095	0.080	0.014
Meramec	0.019**	0.000	0.009	0.527**	0.445**	-0.043
Bourbeuse	0.012**	0.000	0.006	0.428**	0.394**	-0.006
Big	0.014**	0.000	0.009	0.285*	0.230	-0.021
Meramec	0.017**	0.000	0.011	0.311*	0.217	-0.039
Average	0.020	0.000	0.012	0.395	0.273	0.011
Stdev	0.006	0.000	0.004	0.162	0.152	0.041
Missouri Basin						
Yellowstone	0.015**	0.000	0.009	0.250*	0.183	-0.120
Clarks Fork	0.001	0.000	-0.001	0.432**	0.432**	0.051
Yellowstone	0.019**	0.000	0.010	0.410**	0.304*	-0.060
Big Sioux	0.029**	0.000	0.012	0.665**	0.502**	-0.127
North Platte	0.020**	0.000	0.005	0.656**	0.588**	0.061
Bear	0.023**	0.000	0.013*	0.453**	0.308*	-0.055
Elkhorn	0.027**	0.000	0.008	0.704**	0.585**	-0.128
Nisnabottna	0.027**	0.000	0.017**	0.390**	0.129	-0.070
Grand	0.014**	0.000	0.009	0.250*	0.186	0.050
Thompson	0.015**	0.000	0.012*	0.190	0.105	0.017
Gasconade	0.011**	0.000	0.006	0.431**	0.396**	-0.053
Average	0.018	0.000	0.009	0.439	0.338	-0.039
Stdev	0.008	0.000	0.005	0.167	0.168	0.070

Table E-12: Trends and Persistence in Annual 14-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.023**	0.000	0.012	0.692**	0.608**	0.092
Jump	0.027**	0.000	0.019**	0.466**	0.224	-0.005
Black	0.030**	0.000	0.019**	0.472**	0.156	-0.058
Maquaketa	0.018**	0.000	0.009	0.517**	0.440**	0.069
Mississippi	0.022**	0.000	0.010	0.538**	0.417**	-0.025
Rock	0.026**	0.000	0.015*	0.458**	0.216	-0.018
Sugar	0.031**	0.000	0.011	0.701**	0.490**	0.066
Pectonica	0.026**	0.000	0.011	0.579**	0.408**	0.005
Cedar	0.021**	0.000	0.011	0.455**	0.339*	0.056
Skunk	0.014**	0.000	0.010	0.352*	0.285*	0.058
Mississippi	0.027**	0.000	0.013*	0.542**	0.346*	0.008
Des Moines	0.024**	0.000	0.014*	0.534**	0.378**	-0.028
Raccoon	0.020**	0.000	0.017**	0.152	-0.012	-0.004
Iroquois	0.021**	0.000	0.016**	0.290*	0.142	0.015
Kankakee	0.019**	0.000	0.012	0.354*	0.236	-0.026
Spoon	0.014**	0.000	0.011	0.143	0.065	0.016
La Moines	0.006	0.000	0.005	0.172	0.153	0.040
Meramec	0.020**	0.000	0.008	0.596**	0.519**	-0.028
Bourbeuse	0.012**	0.000	0.005	0.500**	0.469**	-0.032
Big	0.016**	0.000	0.009	0.411**	0.340*	-0.028
Meramec	0.017**	0.000	0.009	0.466**	0.392**	-0.064
Average	0.021	0.000	0.012	0.447	0.315	0.005
Stdev	0.006	0.000	0.004	0.154	0.155	0.043
			Missouri Basin			
Yellowstone	0.014**	0.000	0.009	0.181	0.122	-0.080
Clarks Fork	-0.004	0.000	-0.004	0.531**	0.526**	0.081
Yellowstone	0.018**	0.000	0.010	0.369**	0.260*	-0.086
Big Sioux	0.029**	0.000	0.011	0.697**	0.537**	-0.103
North Platte	0.019**	0.000	0.005	0.610**	0.544**	0.020
Bear	0.022**	0.000	0.012*	0.457**	0.329*	-0.046
Elkhorn	0.027**	0.000	0.007	0.730**	0.619**	-0.061
Nisnabottna	0.028**	0.000	0.017**	0.399**	0.113	-0.070
Grand	0.014**	0.000	0.010	0.257*	0.189	0.041
Thompson	0.016**	0.000	0.012*	0.178	0.087	0.006
Gasconade	0.012**	0.000	0.004	0.560**	0.528**	-0.041
Average	0.018	0.000	0.008	0.452	0.350	-0.031
Stdev	0.009	0.000	0.005	0.185	0.195	0.056

Table E-13: Trends and Persistence in Annual 7-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
			Upper Mississippi Basin			
St. Croix	0.023**	0.000	0.012	0.685**	0.599**	0.048
Jump	0.028**	0.000	0.019**	0.501**	0.254*	-0.030
Black	0.031**	0.000	0.020**	0.466**	0.112	-0.088
Maquaketa	0.017**	0.000	0.010	0.483**	0.410**	0.049
Mississippi	0.021	0.000	0.011	0.533**	0.416**	-0.015
Rock	0.026**	0.000	0.016**	0.420**	0.167	-0.020
Sugar	0.032**	0.000	0.012	0.708**	0.482**	0.050
Pectonica	0.027**	0.000	0.011	0.599**	0.418**	-0.002
Cedar	0.021**	0.000	0.010	0.496**	0.382**	0.061
Skunk	0.014**	0.000	0.010	0.349*	0.283*	0.057
Mississippi	0.026**	0.000	0.013*	0.527**	0.341*	0.029
Des Moines	0.025**	0.000	0.014*	0.553**	0.387**	-0.041
Raccoon	0.020**	0.000	0.018**	0.152	-0.017	-0.006
Iroquois	0.023**	0.000	0.017**	0.319*	0.140	0.008
Kankakee	0.019**	0.000	0.011	0.353*	0.243*	-0.019
Spoon	0.014**	0.000	0.011	0.164	0.080	0.019
La Moines	0.008	0.000	0.006	0.216	0.190	0.051
Meramec	0.020**	0.000	0.008	0.619**	0.541**	-0.030
Bourbeuse	0.012	0.000	0.005	0.492**	0.459**	-0.048
Big	0.017**	0.000	0.010	0.438**	0.359*	-0.007
Meramec	0.018**	0.000	0.008	0.491**	0.416**	-0.051
Average	0.021	0.000	0.012	0.455	0.317	0.001
Stdev	0.006	0.000	0.004	0.149	0.157	0.041
			Missouri Basin			
Yellowstone	0.012**	0.000	0.007	0.167	0.119	-0.071
Clarks Fork	-0.006	0.000	-0.005	0.541**	0.533**	0.088
Yellowstone	0.013**	0.000	0.007	0.317*	0.261*	-0.063
Big Sioux	0.029**	0.000	0.011	0.700**	0.541**	-0.092
North Platte	0.018**	0.000	0.005	0.569**	0.505**	-0.022
Bear	0.022**	0.000	0.012*	0.436**	0.308*	-0.028
Elkhorn	0.027**	0.000	0.007	0.757**	0.658**	-0.022
Nisnabottna	0.028**	0.000	0.018**	0.386**	0.101	-0.063
Grand	0.013**	0.000	0.009	0.271*	0.217	0.049
Thompson	0.016**	0.000	0.013*	0.198	0.103	0.004
Gasconade	0.013**	0.000	0.004	0.569**	0.530**	-0.036
Average	0.017	0.000	0.008	0.446	0.352	-0.023
Stdev	0.010	0.000	0.006	0.189	0.197	0.051

Table E-14: Trends and Persistence in Annual 3-Day Low Flows

	Trend			Persistence		
	Obs.	DT	DM	Obs.	DT	DM
Upper Mississippi Basin						
St. Croix	0.026**	0.000	0.013*	0.669**	0.542**	0.070
Jump	0.029**	0.000	0.019**	0.516**	0.262*	-0.014
Black	0.031**	0.000	0.020**	0.458**	0.109	-0.088
Maquaketa	0.017**	0.000	0.010	0.462**	0.378**	0.030
Mississippi	0.019**	0.000	0.011	0.496**	0.393**	-0.003
Rock	0.026**	0.000	0.017**	0.395**	0.137	-0.016
Sugar	0.033**	0.000	0.012	0.721**	0.478**	0.065
Pectonica	0.028**	0.000	0.011	0.622**	0.432**	0.014
Cedar	0.021**	0.000	0.010	0.500**	0.386**	0.056
Skunk	0.017**	0.000	0.011	0.374**	0.288*	0.068
Mississippi	0.024**	0.000	0.013*	0.489**	0.316*	0.033
Des Moines	0.025**	0.000	0.013*	0.573**	0.410**	-0.030
Raccoon	0.021**	0.000	0.018**	0.153**	-0.028	-0.009
Iroquois	0.023**	0.000	0.018**	0.262*	0.060	-0.002
Kankakee	0.020**	0.000	0.013*	0.339*	0.197	-0.048
Spoon	0.014**	0.000	0.012*	0.125	0.040	0.012
La Moines	0.008	0.000	0.006	0.254*	0.226	0.066
Meramec	0.021**	0.000	0.008	0.632	0.553**	-0.028
Bourbeuse	0.013**	0.000	0.006	0.486**	0.448**	-0.048
Big	0.019**	0.000	0.010	0.451**	0.362**	-0.001
Meramec	0.017**	0.000	0.008	0.481**	0.407**	-0.048
Average	0.022	0.000	0.012	0.450	0.304	0.004
Stdev	0.006	0.000	0.004	0.158	0.167	0.045
Missouri Basin						
Yellowstone	0.010*	0.000	0.006	0.167	0.134	-0.063
Clarks Fork	-0.007	0.000	-0.006	0.455**	0.440**	0.025
Yellowstone	0.005	0.000	0.003	0.224	0.211	-0.010
Big Sioux	0.029**	0.000	0.011	0.705**	0.548**	-0.086
North Platte	0.017**	0.000	0.005	0.558**	0.498**	-0.038
Bear	0.022**	0.000	0.013*	0.418**	0.277*	-0.038
Elkhorn	0.028**	0.000	0.007	0.754**	0.643**	-0.041
Nisnabottna	0.027**	0.000	0.017**	0.372**	0.104	-0.045
Grand	0.014**	0.000	0.010	0.300*	0.242*	0.056
Thompson	0.016**	0.000	0.013*	0.203	0.105	0.001
Gasconade	0.014**	0.000	0.004	0.573**	0.534**	-0.036
Average	0.016	0.000	0.008	0.430	0.340	-0.025
Stdev	0.011	0.000	0.006	0.200	0.198	0.040

Effects of De-Trending and De-Markoving

Table E-15: Effect of De-Markoving on Trend at 1%, 5% and >5% Levels of Significance

		Upper Mississippi Basin						*Missouri Basin*			
1-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	1	6	0	7	Obs	1%	0	0	1	1
	5%	0	1	2	3		5%	0	1	3	4
	>5%	0	1	10	11		>5%	0	0	6	6
	Total	1	8	10	21		Total	0	1	10	11
3-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	1	7	1	9	Obs	1%	0	0	2	2
	5%	0	0	0	0		5%	0	0	1	1
	>5%	0	0	12	12		>5%	0	0	8	8
	Total	1	7	13	21		Total	0	0	11	11
7-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	8	1	1	10	Obs	1%	1	0	1	2
	5%	0	0	1	1		5%	0	0	2	2
	>5%	0	0	10	10		>5%	1	0	6	7
	Total	8	1	12	21		Total	2	0	9	11
14-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	2	5	4	11	Obs	1%	1	2	0	3
	5%	0	0	1	1		5%	0	0	3	3
	>5%	0	0	9	9		>5%	0	0	5	5
	Total	2	5	14	21		Total	1	2	8	11
30-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	2	6	5	13	Obs	1%	1	5	0	6
	5%	0	0	0	0		5%	0	0	1	1
	>5%	0	0	8	8		>5%	0	0	4	4
	Total	2	6	13	21		Total	1	5	5	11

Table E-15: Effect of De-Markoving on Trend at 1%, 5% and >5% Levels of Significance (Continued)

		Upper Mississippi Basin						*Missouri Basin*			
60-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	2	5	4	11		1%	3	3	1	7
Obs.	5%	0	0	4	4	Obs	5%	0	0	0	0
	>5%	0	0	6	6		>5%	0	0	4	4
	Total	2	5	14	21		Total	3	3	5	11
90-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	2	5	4	11		1%	1	6	0	7
Obs.	5%	0	0	3	3	Obs	5%	0	0	0	0
	>5%	0	0	7	7		>5%	0	0	4	4
	Total	2	5	14	21		Total	1	6	4	11
180-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	2	5	7	14		1%	1	5	1	7
Obs.	5%	0	0	4	4	Obs	5%	0	1	0	1
	>5%	0	0	3	3		>5%	0	0	3	3
	Total	2	5	14	21		Total	1	6	4	11
Annual Mean Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	1	8	7	16		1%	1	3	3	7
Obs.	5%	0	0	4	4	Obs	5%	0	0	1	1
	>5%	0	0	1	1		>5%	0	0	3	3
	Total	1	8	12	21		Total	1	3	7	11
180-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	1	7	5	13		1%	0	2	4	6
Obs.	5%	0	0	3	3	Obs	5%	0	0	2	2
	>5%	0	0	5	8		>5%	0	0	3	3
	Total	1	7	13	21		Total	0	2	9	11

Table E-15: Effect of De-Markoving on Trend at 1%, 5% and >5% Levels of Significance (Continued)

		Upper Mississippi Basin						*Missouri Basin*			
90-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	2	11	2	15	Obs	1%	0	3	5	8
	5%	0	0	1	1		5%	0	0	1	1
	>5%	0	0	5	5		>5%	0	0	2	2
	Total	2	11	8	21		Total	0	3	8	11
60-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	1	12	4	17	Obs	1%	0	3	3	6
	5%	0	0	2	2		5%	0	0	3	3
	>5%	0	0	2	2		>5%	0	0	2	2
	Total	1	12	8	21		Total	0	3	8	9
30-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	2	7	11	20	Obs	1%	1	2	7	10
	5%	0	0	0	0		5%	0	0	0	0
	>5%	0	0	1	1		>5%	0	0	1	1
	Total	2	7	13	21		Total	1	2	8	11
14-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	4	3	13	20	Obs	1%	1	2	7	10
	5%	0	0	0	0		5%	0	0	0	0
	>5%	0	0	1	1		>5%	0	0	1	1
	Total	4	3	14	21		Total	1	2	8	11
7-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	5	2	11	18	Obs	1%	1	2	7	10
	5%	0	0	0	0		5%	0	0	0	0
	>5%	0	0	3	3		>5%	0	0	1	1
	Total	5	2	14	21		Total	1	2	8	11

Table E-15: Effect of De-Markoving on Trend at 1%, 5% and >5% Levels of Significance (Continued)

		Upper Mississippi Basin						*Missouri Basin*			
3-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	5	5	10	20	Obs	1%	1	2	5	8
	5%	0	0	0	0		5%	0	0	1	1
	>5%	0	0	1	1		>5%	0	0	2	2
	Total	5	5	11	21		Total	1	2	8	11
1-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	5	7	8	20	Obs	1%	1	2	5	8
	5%	0	0	0	0		5%	0	0	0	0
	>5%	0	0	1	1		>5%	0	0	3	3
	Total	5	7	9	21		Total	1	2	8	11

Table E-16: Effect of De-Trending on Persistence at 1%, 5% and >5% Levels of Significance

		Upper Mississippi Basin						*Missouri Basin*			
1-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	0	0	0	0		1%	0	0	0	0
Obs.	5%	0	0	1	1	Obs	5%	0	0	1	1
	>5%	0	0	20	20		>5%	0	0	10	10
	Total	0	0	20	20		Total	0	0	10	10
3-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	0	0	0	0		1%	0	0	0	0
Obs.	5%	0	0	0	0	Obs	5%	0	0	1	1
	>5%	0	0	21	21		>5%	0	0	10	10
	Total	0	0	21	21		Total	0	0	11	11
7-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	0	0	0	0		1%	0	0	0	0
Obs.	5%	0	0	1	1	Obs	5%	0	0	0	0
	>5%	0	0	20	20		>5%	1	0	11	11
	Total	8	1	12	21		Total	2	0	11	11
14-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	0	0	0	0		1%	0	0	1	1
Obs.	5%	0	0	3	3	Obs	5%	0	0	2	2
	>5%	0	0	18	18		>5%	0	0	8	8
	Total	0	0	21	21		Total	0	0	11	11
30-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
	1%	0	0	0	0		1%	0	0	0	0
Obs.	5%	0	0	4	4	Obs	5%	0	0	0	0
	>5%	0	0	17	17		>5%	0	0	11	11
	Total	0	0	21	21		Total	0	0	11	11

Table E-16: Effect of De-Trending on Persistence at 1%, 5% and >5% Levels of Significance (Continued)

Upper Mississippi Basin						*Missouri Basin*					
60-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	0	0	0	0	Obs	1%	0	0	0	0
	5%	0	0	1	1		5%	0	0	0	0
	>5%	0	0	20	20		>5%	0	0	11	11
	Total	0	0	21	21		Total	0	0	11	11
90-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	0	0	0	0	Obs	1%	0	0	0	0
	5%	0	0	4	4		5%	0	0	1	1
	>5%	0	0	17	17		>5%	0	0	10	10
	Total	0	0	21	21		Total	0	0	11	11
180-Day High Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	1	1	0	2	Obs	1%	0	0	1	1
	5%	0	0	7	7		5%	0	0	1	1
	>5%	0	0	12	12		>5%	0	0	9	9
	Total	1	1	19	21		Total	0	0	11	11
Annual Mean Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	1	1	2	4	Obs	1%	0	1	0	1
	5%	0	4	8	12		5%	0	3	2	5
	>5%	0	0	5	5		>5%	0	0	5	5
	Total	1	5	15	21		Total	0	4	7	11
180-Day Low Flows											
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	0	6	1	7	Obs	1%	3	2	0	5
	5%	0	4	4	8		5%	0	1	1	2
	>5%	0	0	6	6		>5%	0	0	4	4
	Total	0	10	11	21		Total	3	3	5	11

Table E-16: Effect of De-Trending on Persistence at 1%, 5% and >5% Levels of Significance (Continued)

		Upper Mississippi Basin						*Missouri Basin*			
		90-Day Low Flows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	2	6	1	9	Obs	1%	4	1	1	6
	5%	0	0	3	3		5%	0	0	2	2
	>5%	0	0	9	9		>5%	0	0	3	3
	Total	2	6	13	21		Total	4	1	6	11
		60-Day Low Flows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	6	4	1	11	Obs	1%	4	2	0	6
	5%	0	0	2	2		5%	0	0	3	3
	>5%	0	0	8	8		>5%	0	0	2	2
	Total	6	4	11	21		Total	4	2	5	11
		30-Day LowFlows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	7	3	2	12	Obs	1%	5	2	1	8
	5%	0	0	6	6		5%	0	0	2	2
	>5%	0	0	3	3		>5%	0	0	1	1
	Total	7	3	11	21		Total	5	2	4	11
		14-Day Low Flows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	9	3	3	15	Obs	1%	5	2	1	8
	5%	0	1	2	3		5%	0	0	1	1
	>5%	0	0	3	3		>5%	0	0	2	2
	Total	9	4	8	21		Total	5	2	4	11
		7-Day Low Flows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	10	3	2	15	Obs	1%	5	1	1	7
	5%	0	2	1	3		5%	0	1	1	2
	>5%	0	0	3	3		>5%	0	0	2	2
	Total	10	5	6	21		Total	5	2	4	11

Table E-16: Effect of De-Trendingon Persistence at 1%, 5% and >5% Levels of Significance (Continued)

		Upper Mississippi Basin						*Missouri Basin*			
		3-Day Low Flows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	11	3	3	17	Obs	1%	5	1	1	7
	5%	0	0	3	3		5%	0	1	0	1
	>5%	0	0	1	1		>5%	0	0	3	3
	Total	11	3	7	21		Total	5	2	4	11
		1-Day Low Flows									
		Residual						Residual			
		1%	5%	>5%	Total			1%	5%	>5%	Total
Obs.	1%	8	6	1	15	Obs	1%	5	1	0	6
	5%	0	0	3	3		5%	0	0	2	2
	>5%	0	0	3	3		>5%	0	0	3	3
	Total	8	6	7	21		Total	5	1	5	11

Schematic Geographic Distribution of Assessed Trends and Persistence

Appendix F: Regional Distributions in Log Space of k-Day Annual High Flows

Upper Mississippi Basin

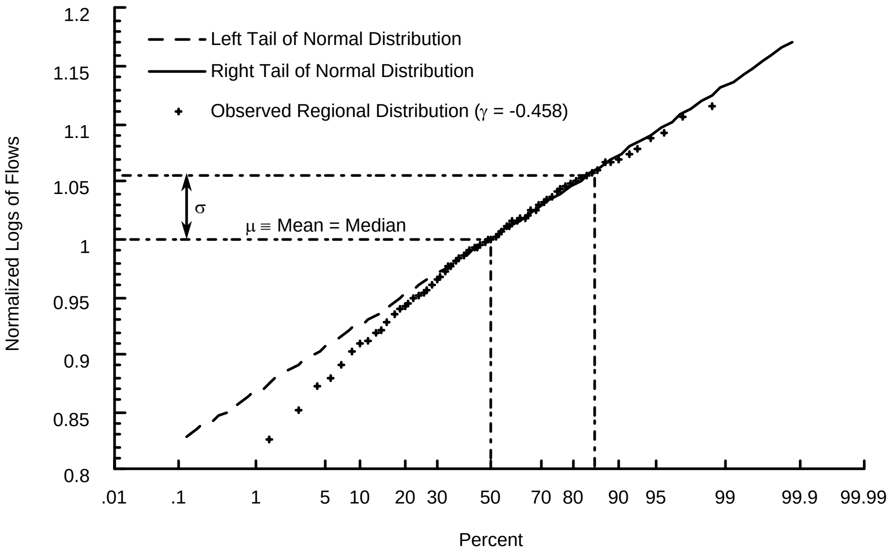


Figure F-1: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 3-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.057$)

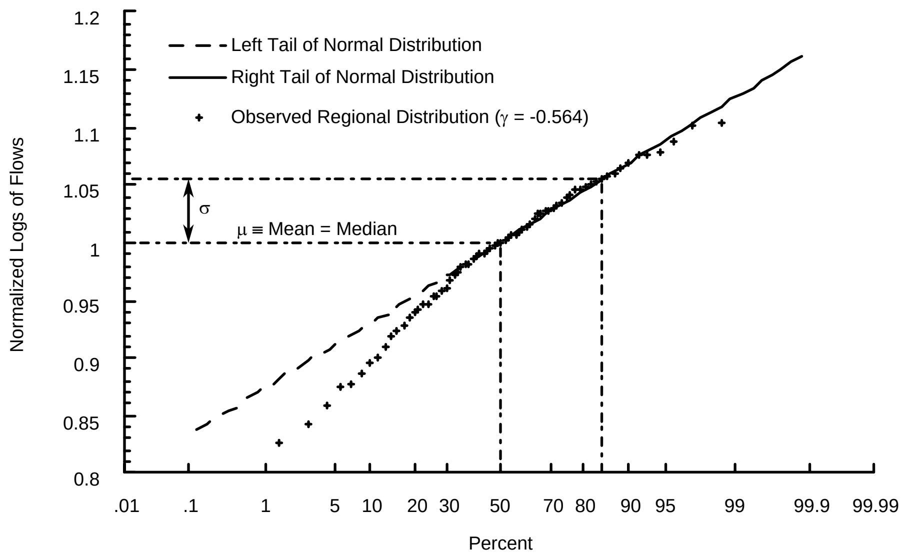


Figure F-2: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 7-Day High Flows with Right Tail fitted with a Normal Distribution (μ = 1, σ = 0.057)

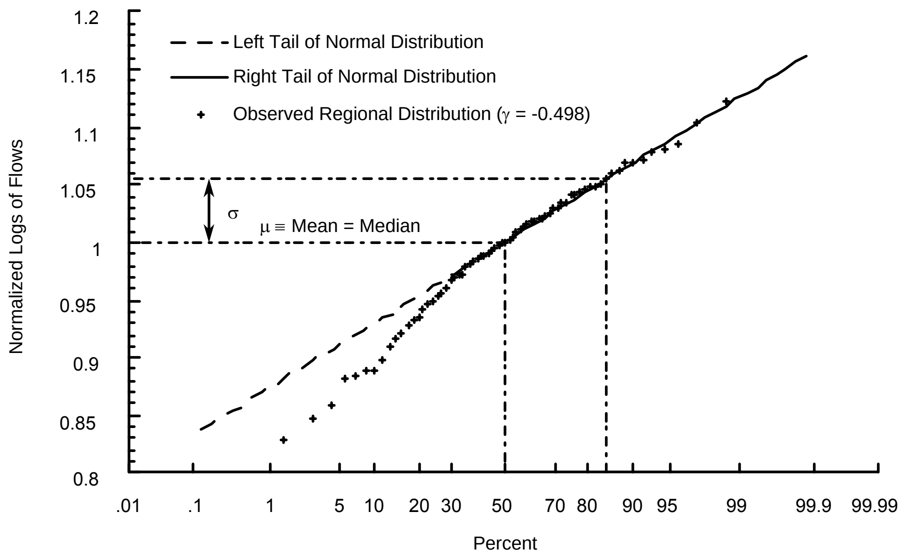


Figure F-3: Upper Mississippi Basin – Regionalized Distribution In Log Space of Annual 14-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 1, σ = 0.054)

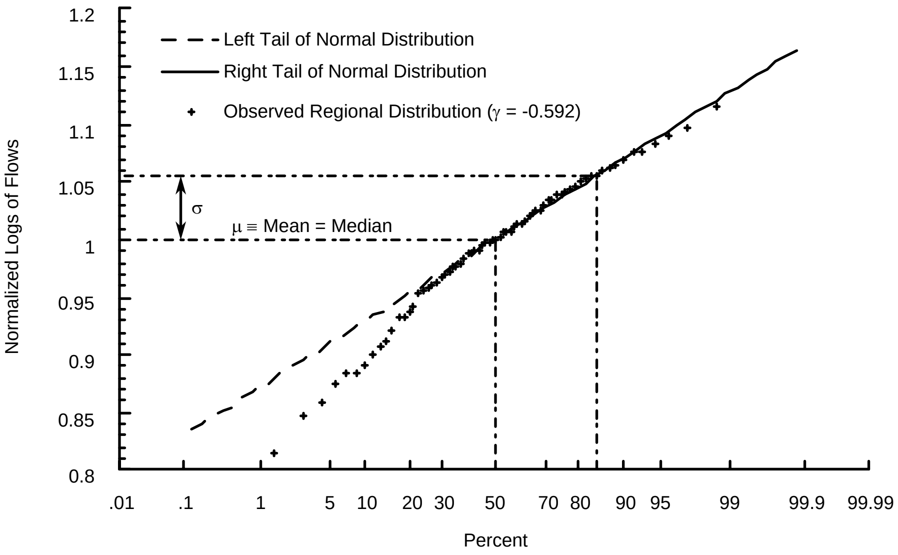


Figure F-4: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 30-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.054$)

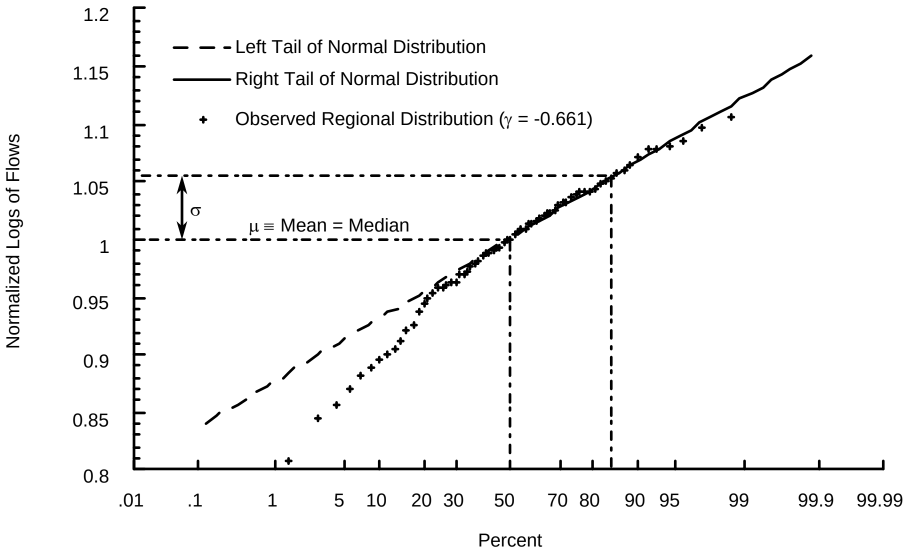


Figure F-5: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 60-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.053$)

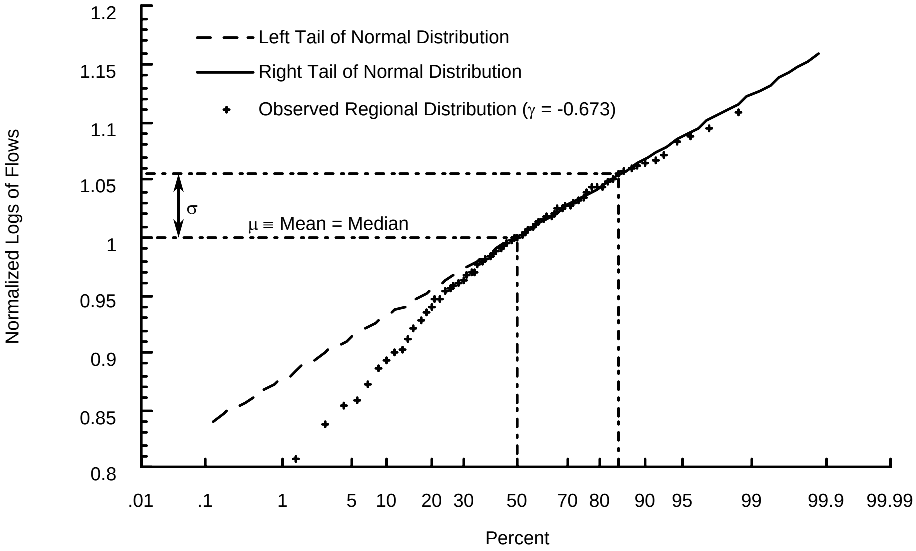


Figure F-6: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 90-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.056$)

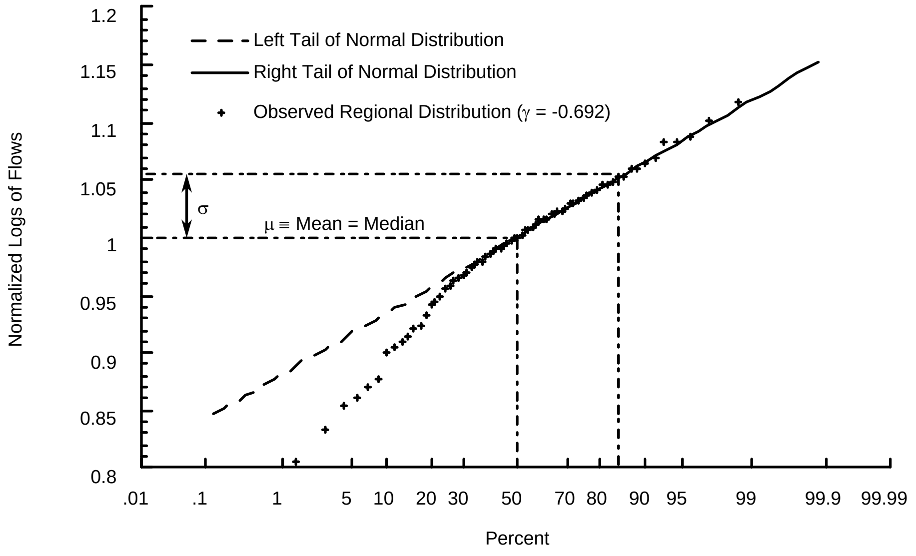


Figure F-7: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 180-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.053$)

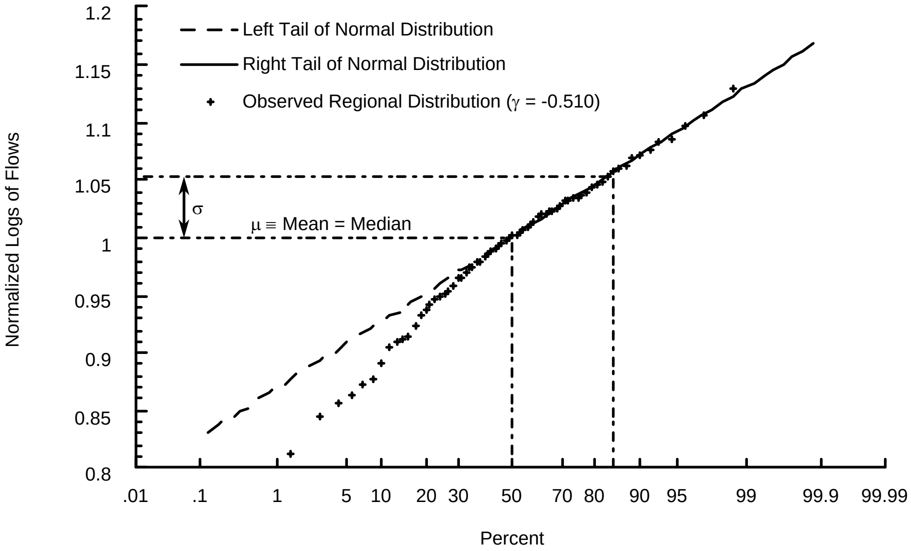


Figure F-8: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual Mean Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.052$)

Missouri Basin

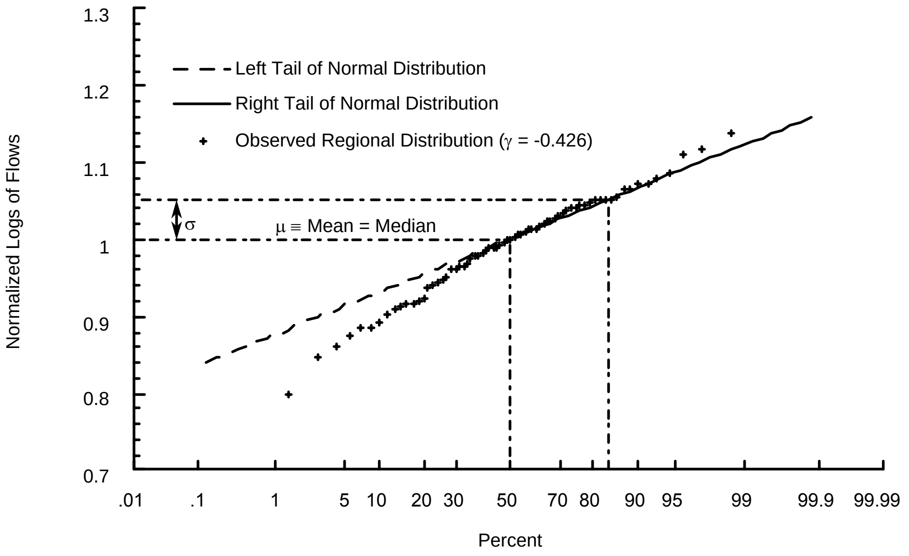


Figure F-9: Missouri Basin – Regionalized Distribution in Log Space of Annual 3-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.052$)

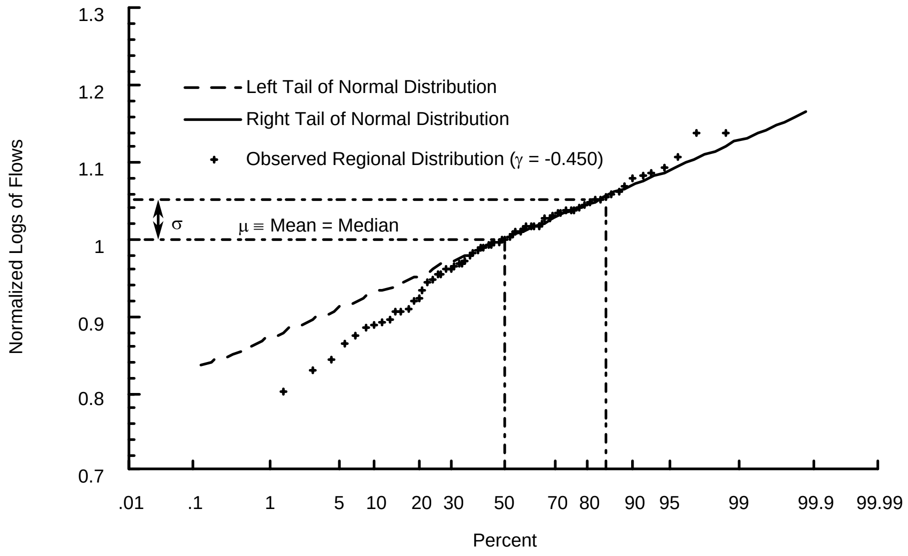


Figure F-10: Missouri Basin – Regionalized Distribution in Log Space of Annual 7-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.053$)

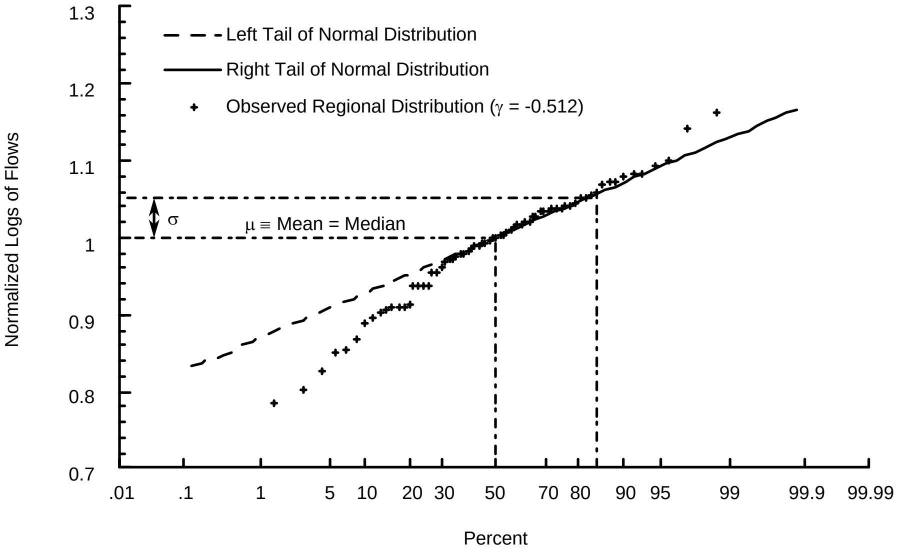


Figure F-11: Missouri Basin – Regionalized Distribution in Log Space of Annual 14-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.057$)

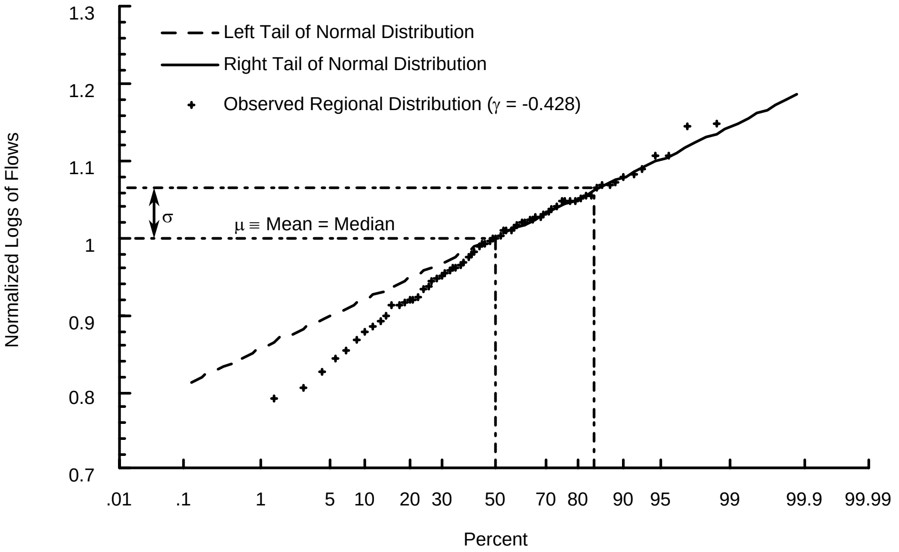


Figure F-12: Missouri Basin – Regionalized Distribution in Log Space of Annual 30-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.063$)

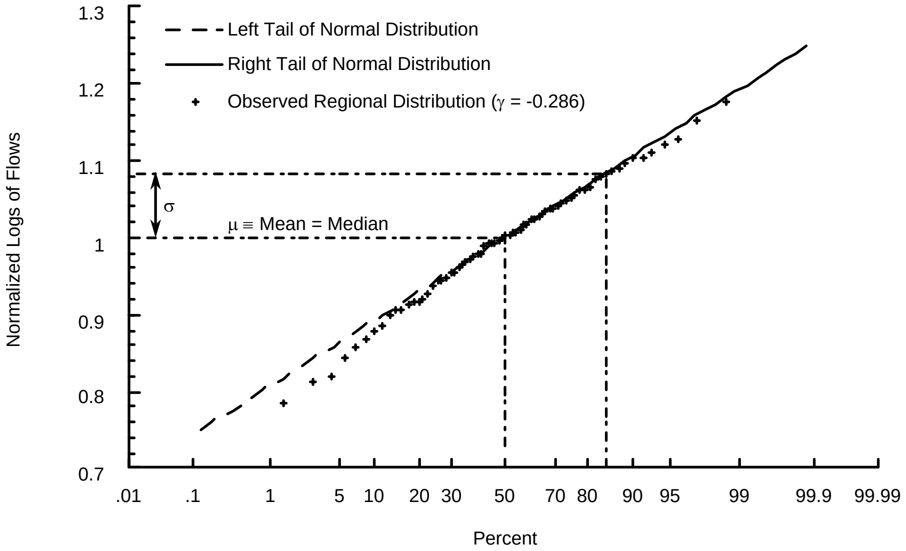


Figure F-13: Missouri Basin – Regionalized Distribution in Log Space of Annual 60-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.083$)

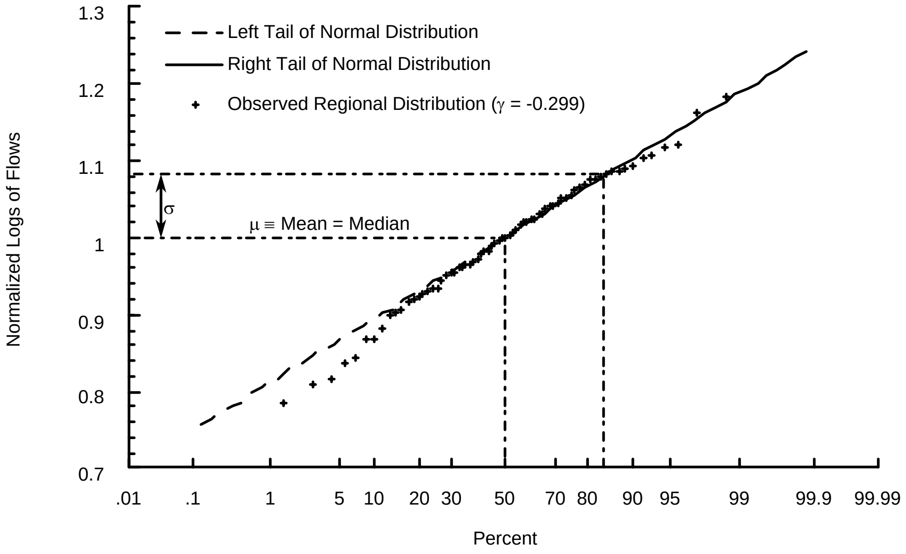


Figure F-14: Missouri Basin – Regionalized Distribution in Log Space of Annual 90-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.081$)

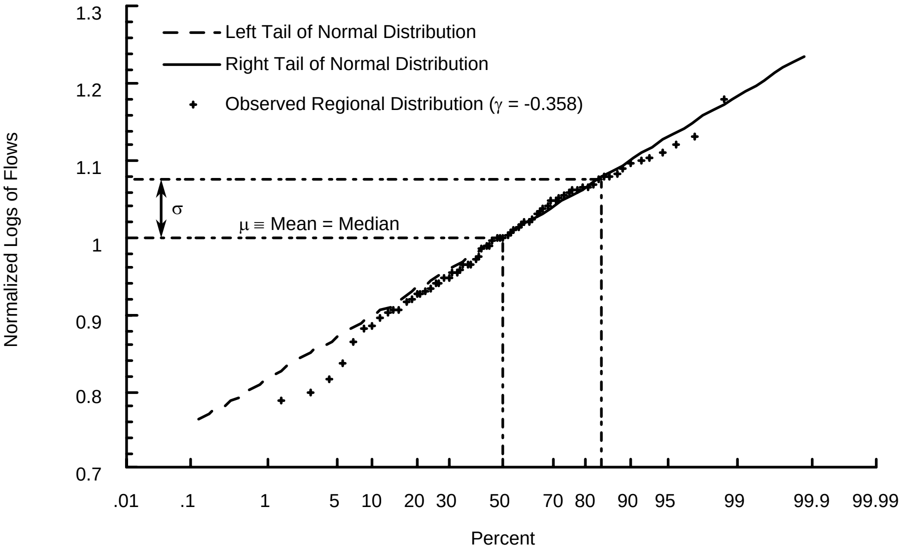


Figure F-15: Missouri Basin – Regionalized Distribution in Log Space of Annual 180-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.079$)

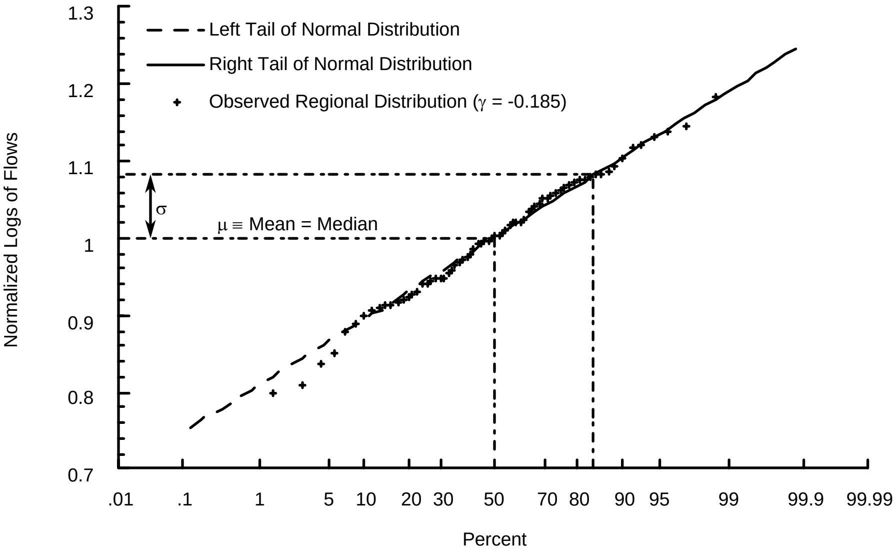


Figure F-16: Missouri Basin – Regionalized Distribution in Log Space of Annual Mean Flows with Right Tail Fitted with a Normal Distribution ($\mu = 1$, $\sigma = 0.083$)

Appendix G: Regional Distributions in Quasi-Log Space of k-Day Annual High Flows

Upper Mississippi Basin

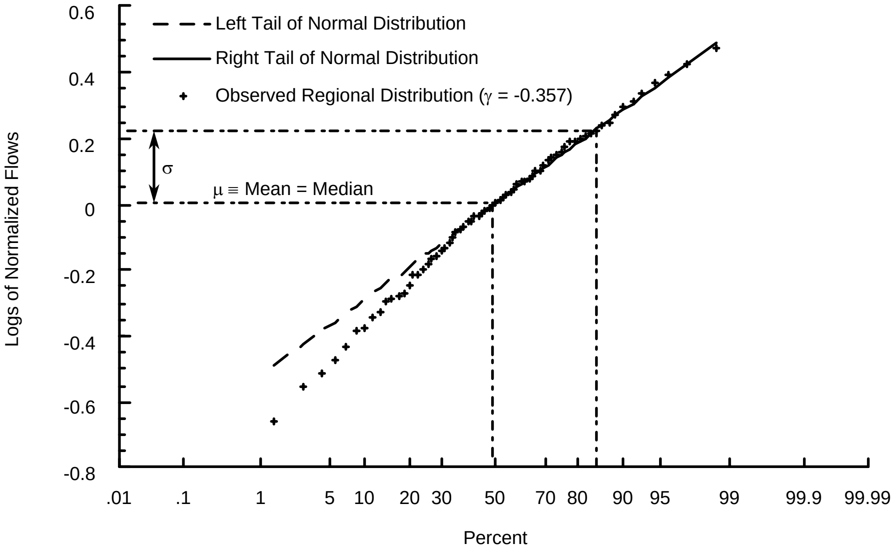


Figure G-1: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 3-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.224$)

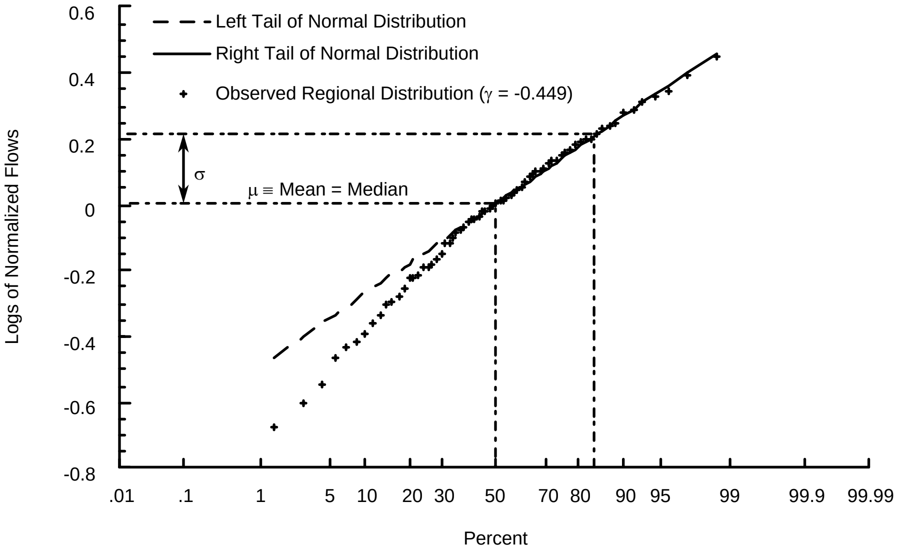


Figure G-2: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 7-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.210)

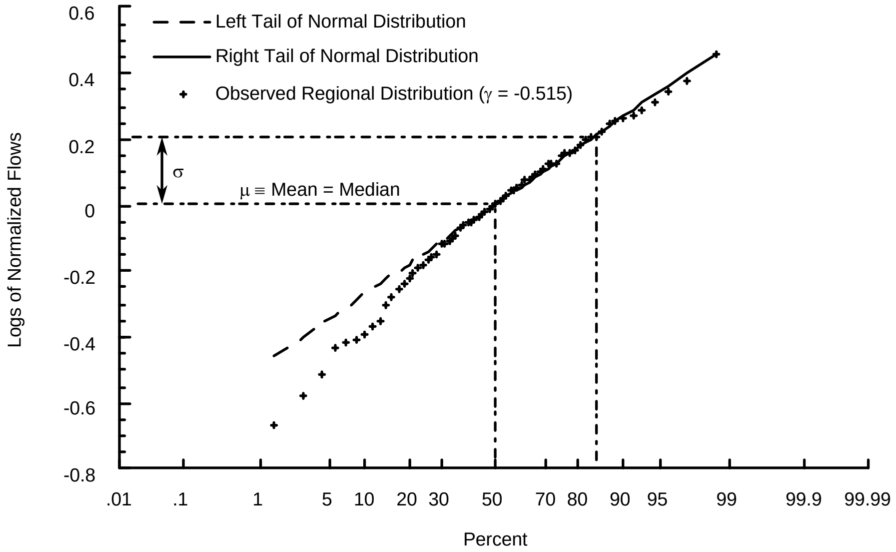


Figure G-3: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 14-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.210)

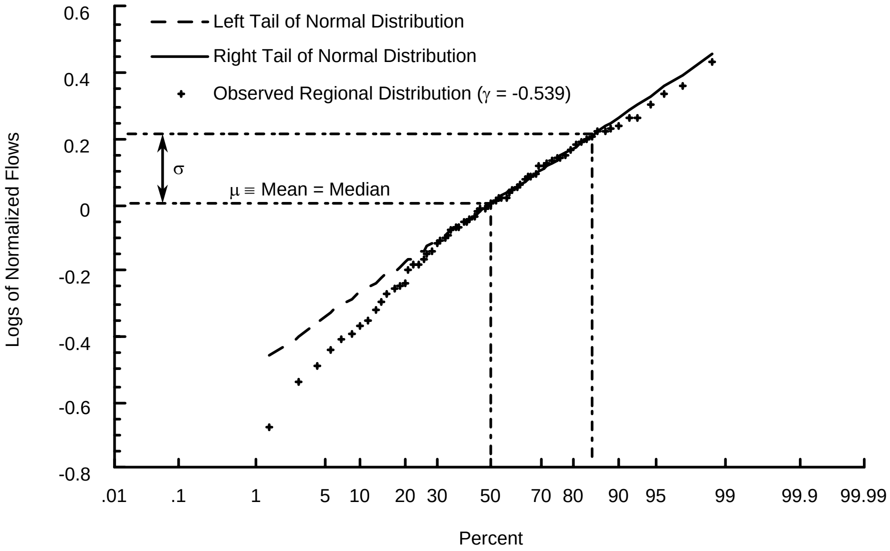


Figure G-4: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 30-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.207)

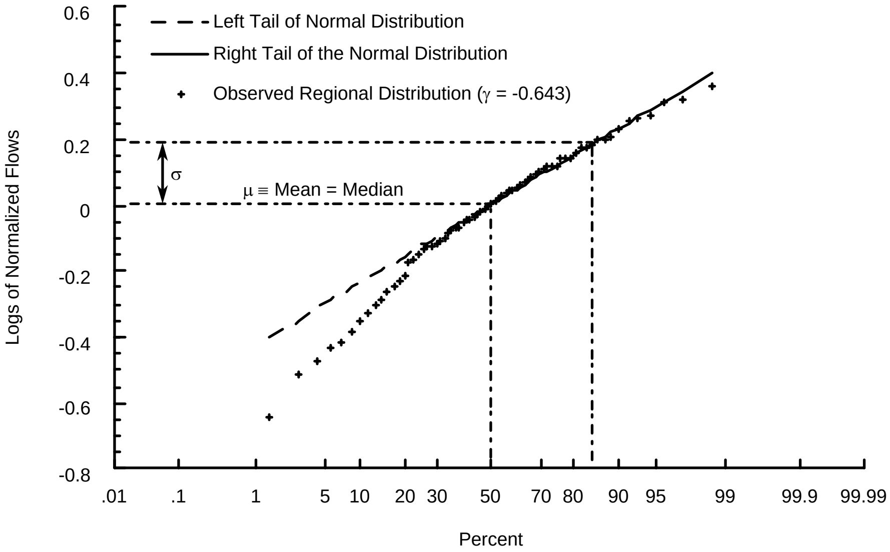


Figure G-5: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 60-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.182)

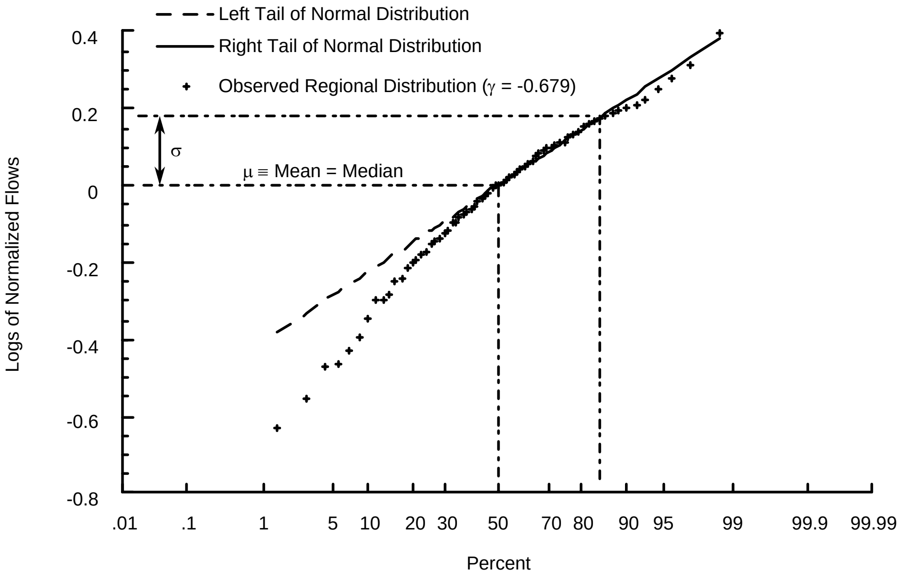


Figure G-6: Upper Mississippi Basin – REgionalized Distribution in Quasi-Log Space of Annual 90-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.174)

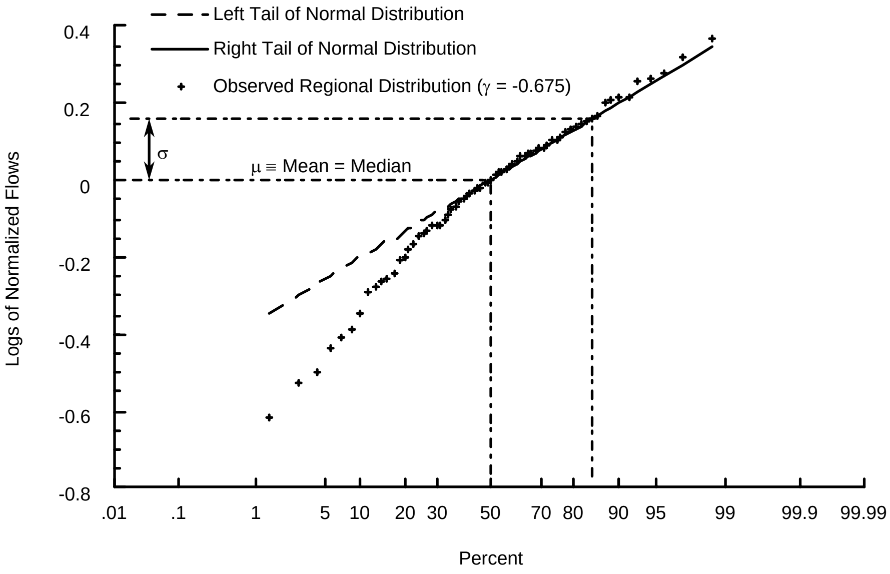


Figure G-7: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 180-Day High Flows with Right Tail Fitted with a Normal Distribution (m = 0, s = 0.157)

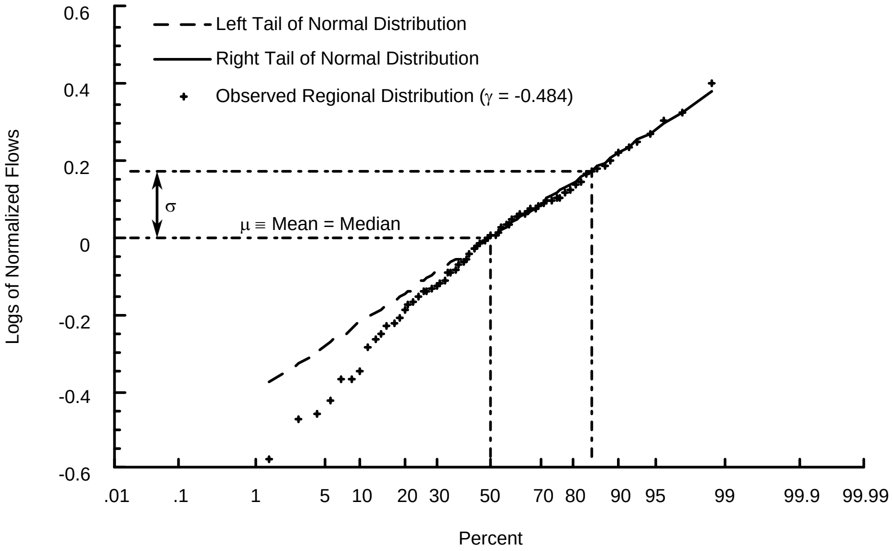


Figure G-8: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual Mean Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.172$)

Missouri Basin

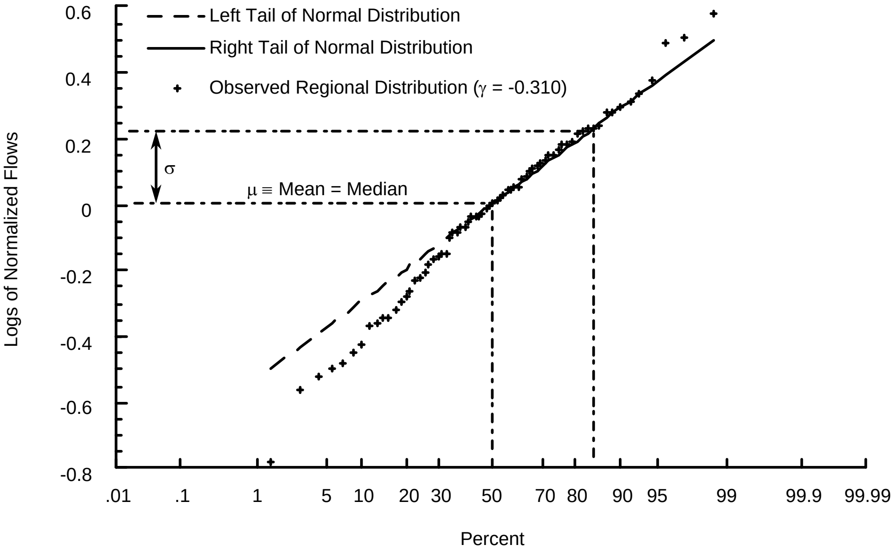


Figure G-9: Missouri Basin – Regionalized Distribution in Quasi-Log Space of 3-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.228$)

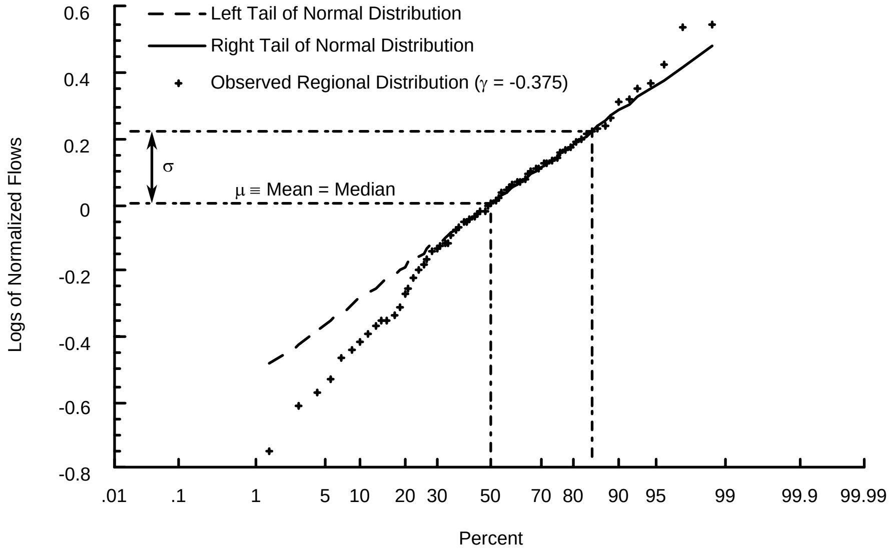


Figure G-10: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 7-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.221$)

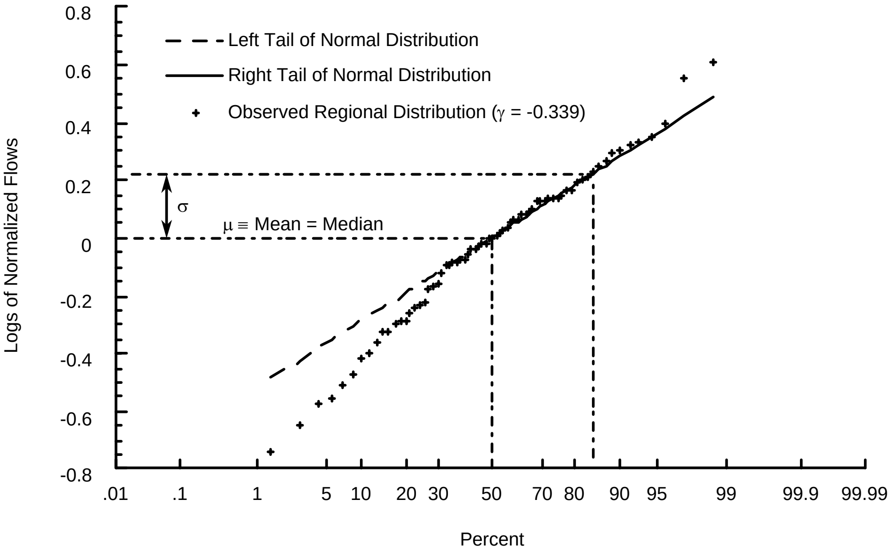


Figure G-11: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 14-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.221$)

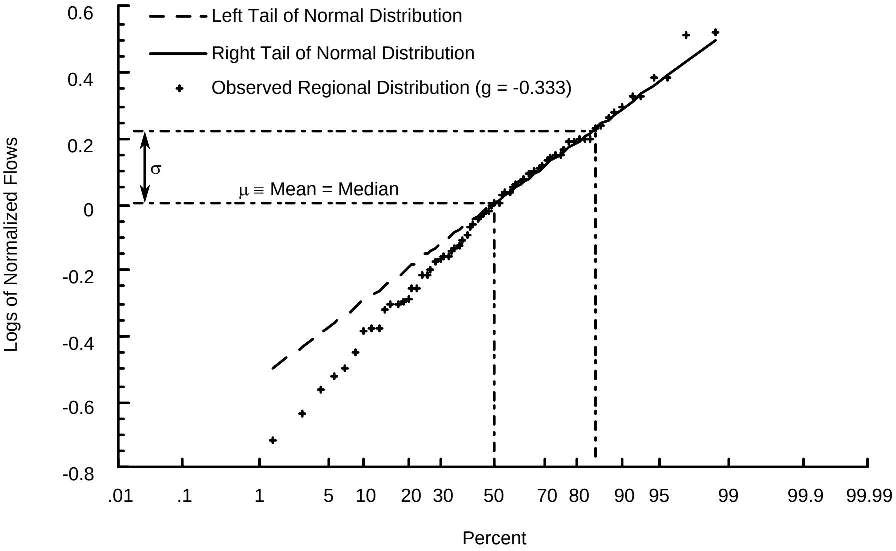


Figure G-12: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 30-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.227)

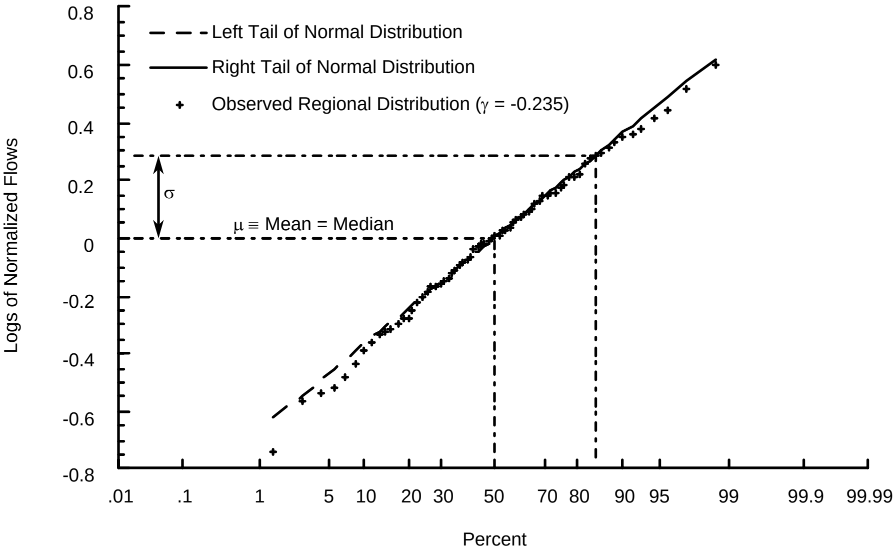


Figure G-13: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 60-Day High Flows with Right Tail Fitted with a Normal Distribution (μ = 0, σ = 0.284)

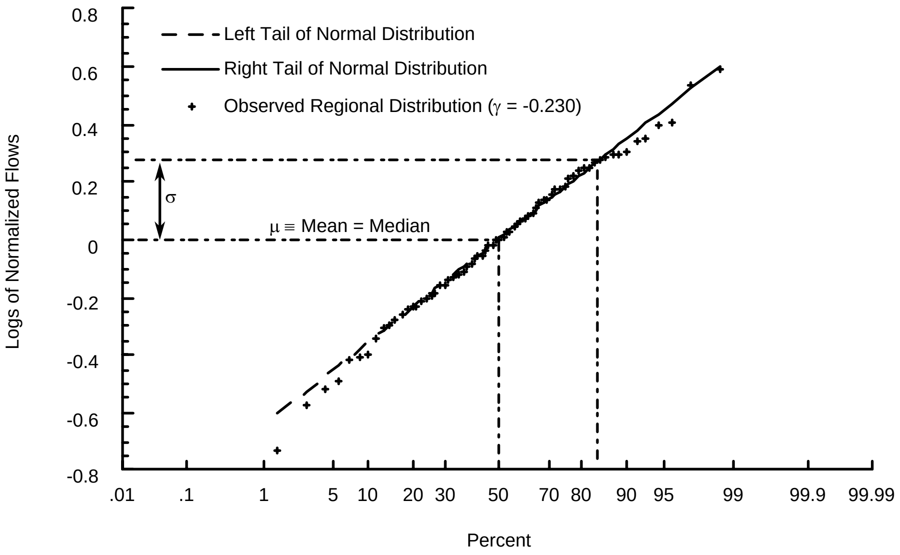


Figure G-14: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 90-Day High Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.274$)

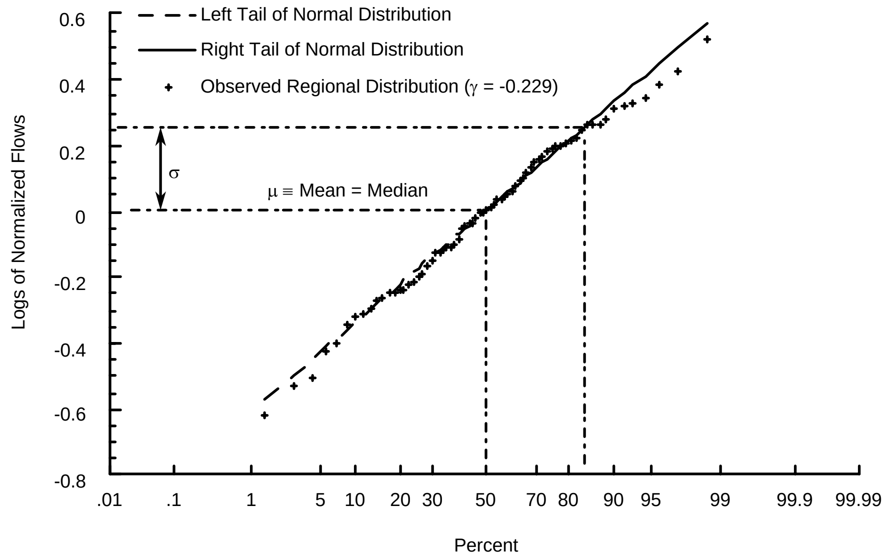


Figure G-15: Missouri Basin – Regionalized Distributions in Quasi-Log Space of Annual 180-Day High Flows with Right Tail Fitted with a Normal Distributioin ($\mu = 0$, $\sigma = 0.260$)

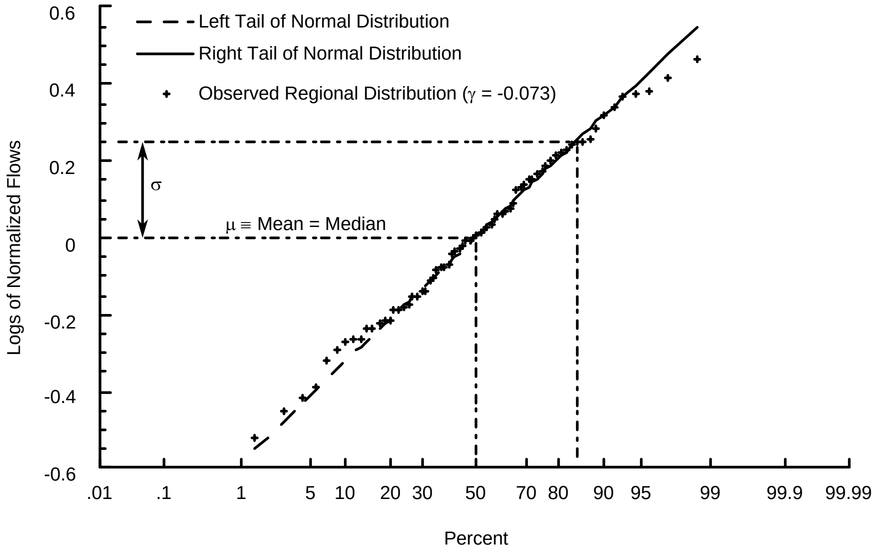


Figure G-16: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual Mean Flows with Right Tail Fitted with a Normal Distribution ($\mu = 0$, $\sigma = 0.249$)

Appendix H: Merged Distributions

A merged distribution is a distribution whose left and right tails are given by two distinct probability laws. In this study, it is presumed that the right tail follows the Normal distribution, and the left tail the Weibull distribution. Although the Pearson Type III distribution might be of greater hydrologic interest in describing the left tail, the Weibull distribution is more analytically tractable as it can be expressed explicitly in the form of its cumulative distribution function, its probability density function or its inverse function. As the purpose here is to examine how the tails interact, then it is sufficient to consider for the left tail a distribution that admits both positive and negative skews. The merged distribution is referred to as the Weibull/Normal distribution. A complete Weibull (Normal) distribution is a merged distribution where both tails follow a Weibull (Normal) distribution.

Two merged distributions are considered. For both, the right tail follows the Normal distribution. For one of the merged distributions, the left tail follows a Weibull distribution that is bounded below, and for the other merged distribution, the left tail follows a Weibull distribution that is unbounded below. The latter Weibull distribution is what is often termed the Type 3 extreme value distribution.

Complete Weibull and Normal Distributions

The complete Weibull distribution for $x \geq m$ is bounded below and may be expressed in its cumulative and density forms as

$$F(x) = 1 - exp\left[-\left(\frac{x-m}{a}\right)^{b}\right] \tag{H-1}$$

$$f(x) = \frac{b}{a}\left(\frac{x-m}{a}\right)^{b-1} exp\left[-\left(\frac{x-m}{a}\right)^{b}\right] \tag{H-2}$$

where the parameters of location, m, scale, $a > 0$ and shape, $b > 0$ may be expressed in terms of the mean, μ_W, variance, σ_W^2, and skewness, γ_W,, as well as the kurtosis, λ_W, as

$$\mu_W = m + a\Gamma(1 + 1/b) \tag{H-3}$$

$$\sigma_W^2 = a\left\{\Gamma(1+2/b) - \Gamma^2(1+1/b)\right\} \tag{H-4}$$

$$\gamma_W = \frac{\left\{\Gamma(1+3/b) - 3\Gamma(1+2/b)\Gamma(1+1/b) + 2\Gamma^3(1+1/b)\right\}}{\left\{\Gamma(1+2/b) - \Gamma^2(1+1/b)\right\}^{3/2}} \tag{H-5}$$

$$\lambda_W = \frac{\left\{\begin{array}{l} \Gamma(1+4/b) - 4\Gamma(1+3/b)\Gamma(1+1/b) + 6\Gamma(1+2/b)\Gamma^2(1+1/b) \\ \Gamma^4(1+1/b) \end{array}\right\}}{\left\{\Gamma(1+2/b) - \Gamma(1+1/b)\right\}^2} \tag{H-6}$$

The skewness, γ_W, and the kurtosis, λ_W, are functions of only the shape parameter, b. As $b \rightarrow \infty$, the probability meass tends very rapidly to become concenetrated at the modal value

$$x' = m + a[1 - 1/b]^{1/b} \tag{H-7}$$

The median is given by the solution of $F(x) = 0.5$,

$$\begin{aligned} \tilde{x} &= m + a[-ln(0.5)]^{1/b} \\ &= m + a[0.693\cdots]^{1/b} \end{aligned} \tag{H-8}$$

For $x \leq m$, the complete Weibull distribution is unbounded below and may be expressed in its cumulative and density forms as

$$F(x) = exp\left[-\left(\frac{m-x}{a}\right)^b\right] \tag{H-9}$$

$$f(x) = \frac{b}{a}\left(\frac{m-x}{a}\right)^{b-1} exp\left[-\left(\frac{m-x}{a}\right)^b\right] \tag{H-10}$$

where the parameters of location, m, scale, $a > 0$ and shape, $b > 0$ may be expressed in terms of the mean, μ_W, variance, σ_W^2, and skewness, γ_W,, as well as the kurtosis, λ_W, as

$$\mu_W = m - a\Gamma(1+1/b) \tag{H-11}$$

$$\sigma_W^2 = a\left\{\Gamma(1+2/b) - \Gamma^2(1+1/b)\right\} \qquad \text{(H-12)}$$

$$\gamma_W = -\frac{\left\{\Gamma(1+3/b) - 3\Gamma(1+2/b)\Gamma(1+1/b) + 2\Gamma^3(1+1/b)\right\}}{\left\{\Gamma(1+2/b) - \Gamma^2(1+1/b)\right\}^{3/2}} \qquad \text{(H-13)}$$

$$\lambda_W = \frac{\left\{\begin{matrix}\Gamma(1+4/b) - 4\Gamma(1+3/b)\Gamma(1+1/b) + 6\Gamma(1+2/b)\Gamma^2(1+1/b) \\ \Gamma^4(1+1/b)\end{matrix}\right\}}{\left\{\Gamma(1+2/b) - \Gamma(1+1/b)\right\}^2} \qquad \text{(H-14)}$$

For the complete Weibull unbounded below, the modal and median values are given by

$$x'' = m - a\left[1 - 1/b\right]^{1/b} \qquad \text{(H-15)}$$

$$\begin{aligned}\breve{x} &= m - a\left[-\ln(0.5)\right]^{1/b} \\ &= m - a\left[0.693\cdots\right]^{1/b}\end{aligned} \qquad \text{(H-16)}$$

The Normal distribution can be expressed explicitly only in its density form,

$$f(x) = \frac{1}{\sqrt{2\pi}d} exp\left[-\frac{1}{2}\left(\frac{x-c}{d}\right)^2\right] - \infty \le x \le \infty \qquad \text{(H-17)}$$

where the location parameter, c, and the scale parameter, d, in terms of the mean and varirance are simply

$$\mu_N = c \qquad \text{(H-18)}$$

$$\sigma^2 = d^2 \qquad \text{(H-19)}$$

The skewness and kurtosis are constant, namely

$$\gamma_N = 0 \qquad \text{(H-20)}$$

$$\lambda_N = 0 \qquad \text{(H-21)}$$

For the complete Normal distribution, the modal and median values are equal to the mean.

Merged Weibull and Normal Distributions

For a merged Weibull/Normal distribution, where the left tail follows a Weibull distribution bounded below and the right tail follows a Normal distribution, the cumulative and density forms are

$$F(x)=\begin{cases} 1-exp\left[-\left(\frac{x-m}{a}\right)^{b}\right]; m<x<\tilde{x} \\ \int_{\tilde{x}}^{x}\frac{1}{\sqrt{2\pi}d}exp\left[-\frac{1}{2}\left(\frac{x-c}{d}\right)^{2}\right]; x\geq c\equiv\tilde{x} \end{cases} \qquad \text{(H-22)}$$

$$f(x)=\begin{cases} \frac{b}{a}\left(\frac{x-m}{a}\right)^{b-1} exp\left[-\left(\frac{x-m}{a}\right)^{b}\right]; m<x<\tilde{x} \\ \frac{1}{\sqrt{2\pi}d}exp\left[-\frac{1}{2}\left(\frac{x-c}{d}\right)^{2}\right]; x\geq c\equiv\tilde{x} \end{cases} \qquad \text{(H-23)}$$

where $\tilde{x}$ is given by Eq. (H-8).

For a merged Weibull/Normal distribution, where the left tail follows a Weibull distribution unbounded below and the right tail follows a Normal distribution, the cumulative and density forms are

$$F(x)=\begin{cases} exp\left[-\left(\frac{m-x}{a}\right)^{b}\right]; m>x<\breve{x} \\ \int_{\breve{x}}^{x}\frac{1}{\sqrt{2\pi}d}exp\left[-\frac{1}{2}\left(\frac{x-c}{d}\right)^{2}\right]; x\geq\breve{x}\equiv c \end{cases} \qquad \text{(H-24)}$$

$$f(x)=\begin{cases} \frac{b}{a}\left(\frac{m-x}{a}\right)^{b-1} exp\left[-\left(\frac{m-x}{a}\right)^{b}\right]; m>x<\breve{x} \\ \frac{1}{\sqrt{2\pi}d}exp\left[-\frac{1}{2}\left(\frac{y-c}{d}\right)^{2}\right]; x\geq\breve{x}\equiv c \end{cases} \qquad \text{(H-25)}$$

where $\breve{x}$ is given by Eq. (H-13).

The merged Weibull/Normal distributions are defined by 5 parameters, namely, m, a, b, c and d. The parameters c and d are related to the parameters m, a and b. The left and right tails are joined at the median, whereby,

$$\begin{aligned} c &= \tilde{x} \\ &= m + a[0.693\cdots]^{1/b} \end{aligned} \tag{H-26}$$

for the merged Weibull/Normal distribution bound below, and

$$\begin{aligned} c &= \breve{x} \\ &= m - a[0.693\cdots]^{1/b} \end{aligned} \tag{H-27}$$

for the merged Weibull/Normal distribution unbound below. It is presumed that there is no discontinuity in the tails at the median, whereby, for the merged Weibull/Normal distribution bound below,

$$f_W(\tilde{x}) = f_N(\tilde{x}) \tag{H-28}$$

It follows that

$$\begin{aligned} \frac{b}{a}\left(\frac{\tilde{x}-m}{a}\right)^{b-1} exp\left[-\left(\frac{\tilde{x}-m}{a}\right)^{b}\right] &= \frac{1}{\sqrt{2\pi}d} exp\left[-\frac{1}{2}\left(\frac{\tilde{x}-c}{d}\right)^{2}\right] \\ &= \frac{1}{\sqrt{2\pi}d} \end{aligned} \tag{H-29}$$

whereby,

$$d = \frac{a}{\sqrt{2\pi}b}\left(\frac{\tilde{x}-m}{a}\right)^{1-b} exp\left[\left(\frac{\tilde{x}-m}{a}\right)^{b}\right] \tag{H-30}$$

For the merged Weibull/Normal unbound below

$$f_W(\breve{x}) = f_N(\breve{x}) \tag{H-31}$$

whereby,

$$d = \frac{a}{\sqrt{2\pi b}} \left(\frac{m - \breve{x}}{a} \right)^{1-b} exp \left[\left(\frac{m - \breve{x}}{a} \right)^{b} \right] \tag{H-32}$$

The relations between the parameters and the statistical characteristics, namely, the mean, $\mu_{W|N}$, variance, $\sigma^2_{W|N}$, skewness, $\gamma_{W|N}$, and kurtosis, $\lambda_{W|N}$, can not be expressed explicitly. For $m = 0$ and $a = 1$, values of mean, variance, skewness and kurtosis for specific values of b are given in Table H-1 for the complete Weibull and merged Weibull/Normal distributions that are bounded below, and in Table H-2 for the complete Weibull and merged Weibull/Normal distributions that are unbounded below.

Table H-1: Statistical Characteristics for Specific Values of b – Distributions Bounded Below

b	μ	σ^2	γ	λ
		Complete Weibull		
3.59	0.90	0.08	0.00	2.72
3.40	0.09	0.08	0.05	2.71
3.21	0.90	0.09	0.10	2.71
2.90	0.89	0.11	0.02	2.74
2.76	0.89	0.12	0.25	2.76
2.21	0.89	0.18	0.50	3.02
1.83	0.89	0.25	0.75	3.48
1.55	0.90	0.35	1.00	4.16
1.36	0.92	0.46	1.25	4.98
1.21	0.94	0.61	1.50	6.02
0.99	1.00	1.03	2.00	8.87
		Merged Weibull/Normal		
4.23	0.92	0.06	0.00	2.90
3.84	0.91	0.07	0.05	2.86
3.49	0.90	0.08	0.10	2.82
2.92	0.89	0.11	0.20	2.77
2.67	0.88	0.12	0.25	2.76
1.74	0.86	0.22	0.50	2.81
1.12	0.83	0.36	0.75	3.05
0.68	0.79	0.53	1.00	3.48
0.36	0.68	0.61	1.25	4.08
0.11	0.17	0.06	1.50	4.86

Table H-2: Statistical Characteristics for Specific Values of b – Distributions Unbounded Below

b	μ	σ^2	γ	λ
		Complete Weibull		
3.59	-0.90	0.08	-0.00	2.72
3.40	-0.90	0.08	-0.05	2.71
3.21	-0.90	0.09	-0.10	2.71
2.90	-0.89	0.11	-0.20	2.74
2.76	-0.89	0.12	-0.25	2.76
2.21	-0.89	0.18	-0.50	3.02
1.83	-0.89	0.25	-0.75	3.48
1.55	-0.90	0.35	-1.00	4.16
1.36	-0.92	0.46	-1.25	4.98
1.21	-0.94	0.61	-1.50	6.02
0.99	-1.00	1.03	-2.00	8.87
		Merged Weibull/Normal		
2.88	-0.88	0.12	0.00	2.96
2.60	-0.87	0.15	-0.05	3.01
2.38	-0.87	0.18	-0.10	3.06
2.06	-0.86	0.25	-0.20	3.19
1.93	-0.85	0.28	-0.25	3.27
1.51	-0.84	0.48	-0.50	3.76
1.27	-0.84	0.70	-0.75	4.44
1.12	-0.85	0.96	-1.00	5.28
1.01	-0.87	1.25	-1.25	6.32
0.93	-0.90	1.57	-1.50	7.50
0.81	-0.96	2.39	-2.00	10.60

Eqs. (H-22) and (H-23) define merged Weibull/Normal distributions that are right tail Normal and left tail sub-Normal. See Figures H-1, H-2 and H-3. Eqs. (H-24) and (H-25) define a merged Weibull/Normal distributions that are right tail Normal and left tail super-Normal. See Figures H-4, H-5 and H-6.

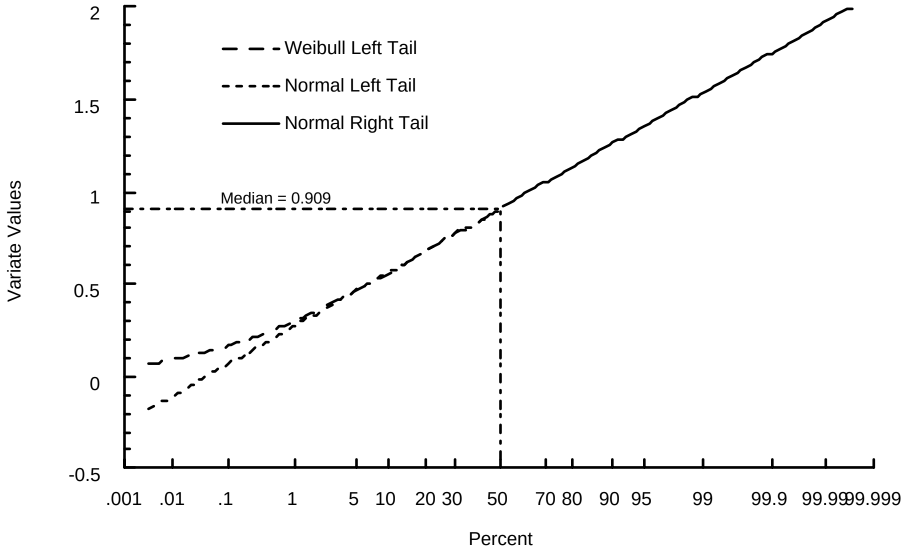


Figure H-1: Normal Distribution (γ=0) Relative to a Weibull/Normal Distribution Bound Below (γ=0.05)

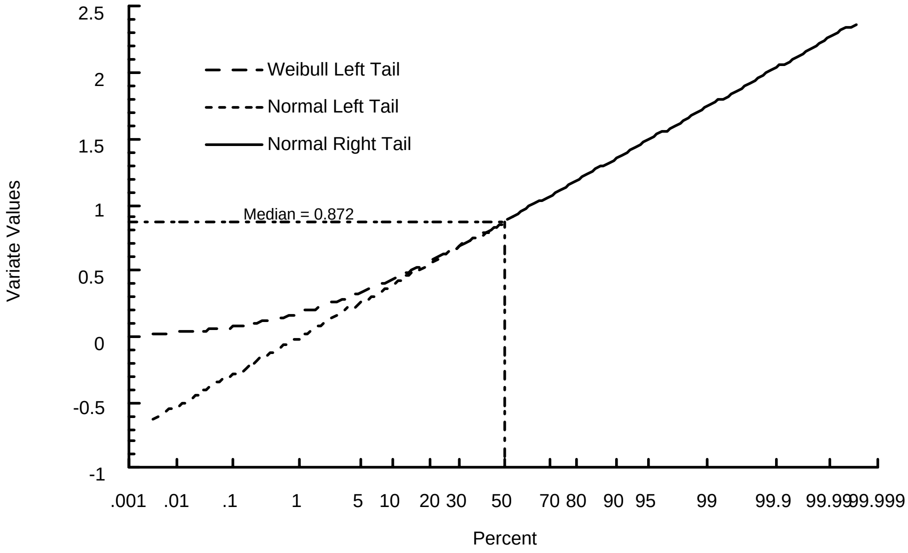


Figure H-2: Normal Distribution (γ=0) Relative to a Weibull/Normal Distribution Bound Below (γ=0.25)

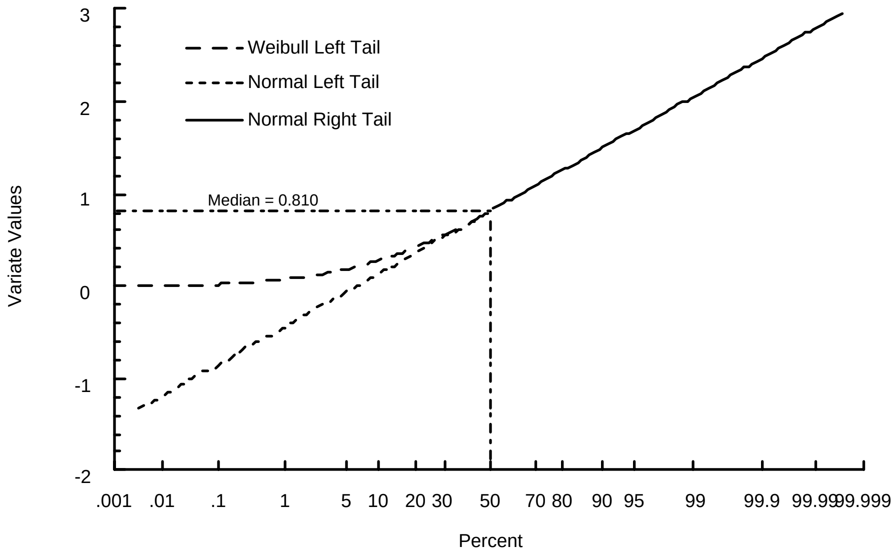


Figure H-3: Normal Distribution (γ=0) Relative to a Weibull/Normal Distribution Bound Below (γ=0.50)

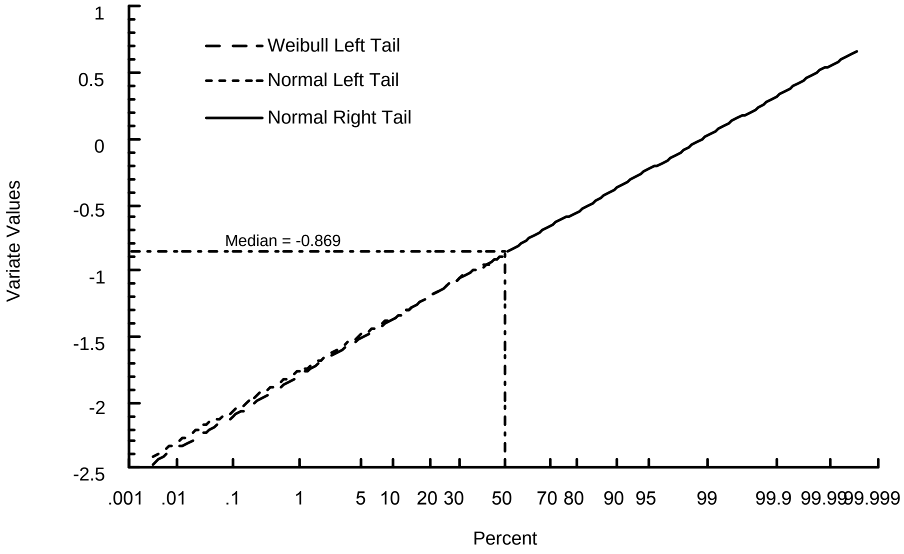


Figure H-4: Normal Distribution (γ=0) Relative to a Weibull/Normal Distribution Unbound Below (γ=−0.05)

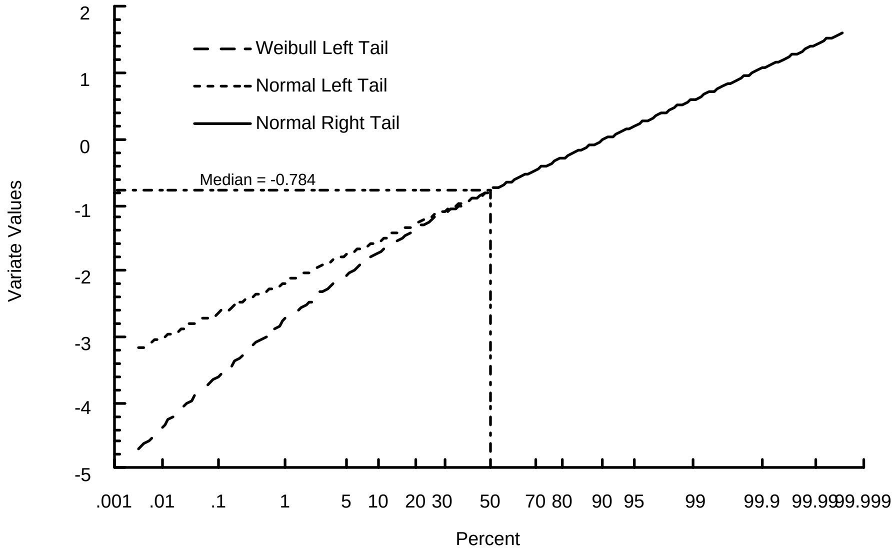


Figure H-5: Normal Distribution (γ=0) Relative to a Weibull/Normal Distribution Unbound Below (γ=−0.50)

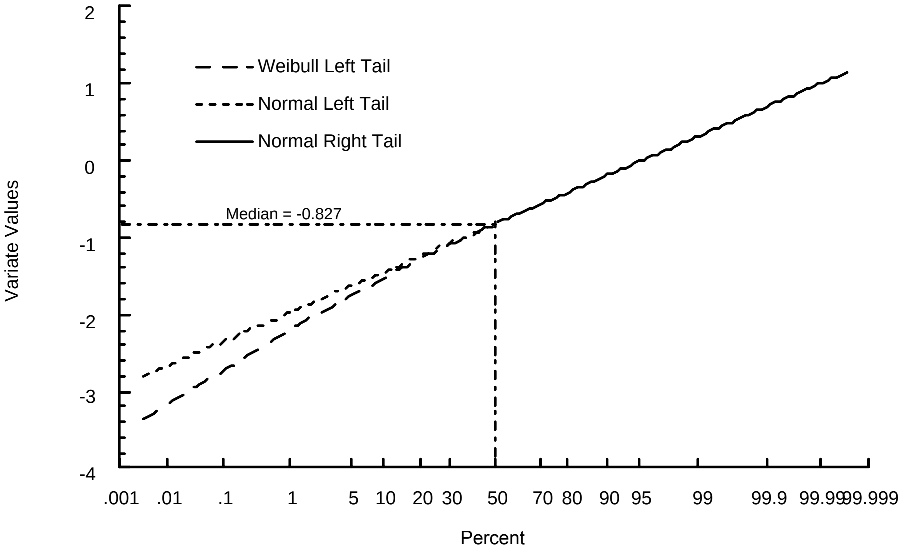


Figure H-6: Normal Distribution (γ=0) Relative to a Weibull/Normal Right Distribution Unbound Belov (γ=−0.25)

Appendix I: At-Site Distributions of Sequences of Annual 1-Day High Flows in Log Space Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

The fit of a Right-Tail Normal distribution is made with only one of the three methods for determining the parameter σ, namely, the mirrored spread method. Only the right tail of the Right-Tail Normal distribution is shown. Both tails of the fitted Pearson Type III distribution are shown.

Upper Mississippi Basin

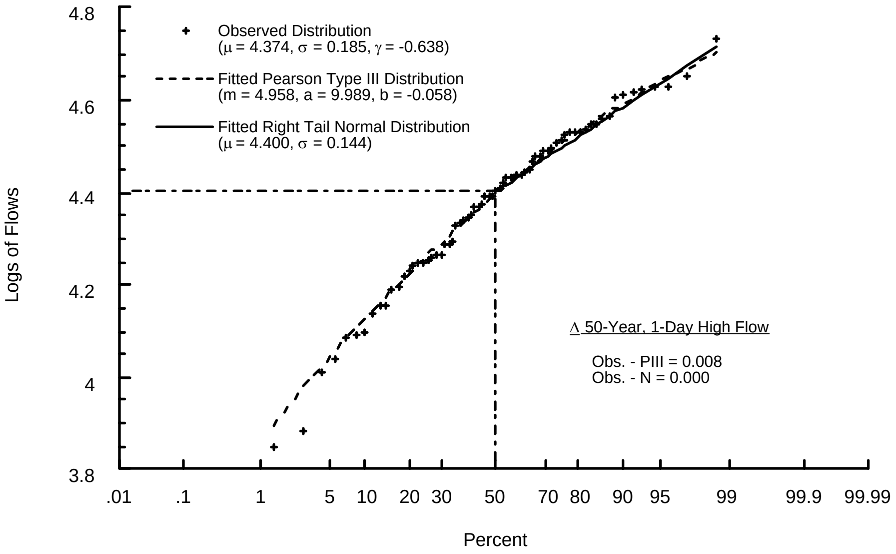


Figure I-1: Upper Mississippi Basin (St. Croix at St. Croix, WI) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

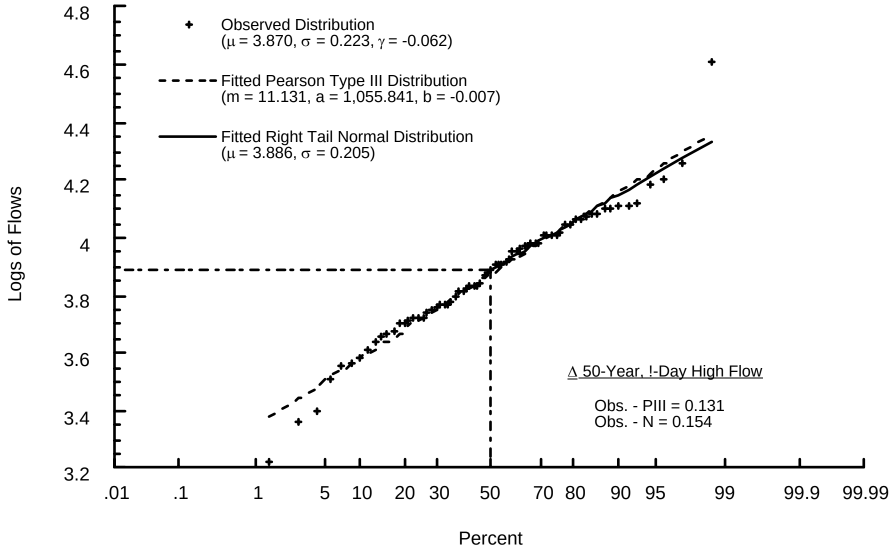


Figure I-2: Upper Mississippi Basin (Jump at Sheldon, WI) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

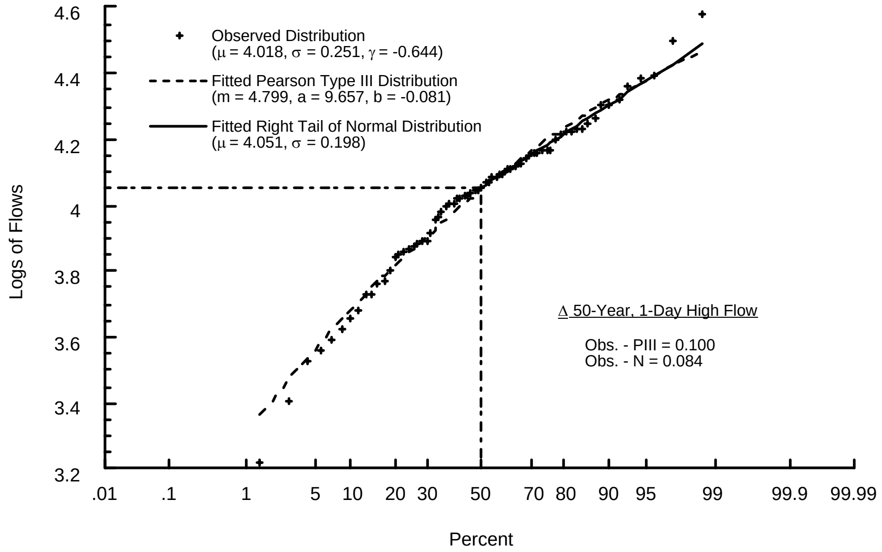


Figure I-3: Upper Mississippi Basin (Black at Sheldon, WI) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

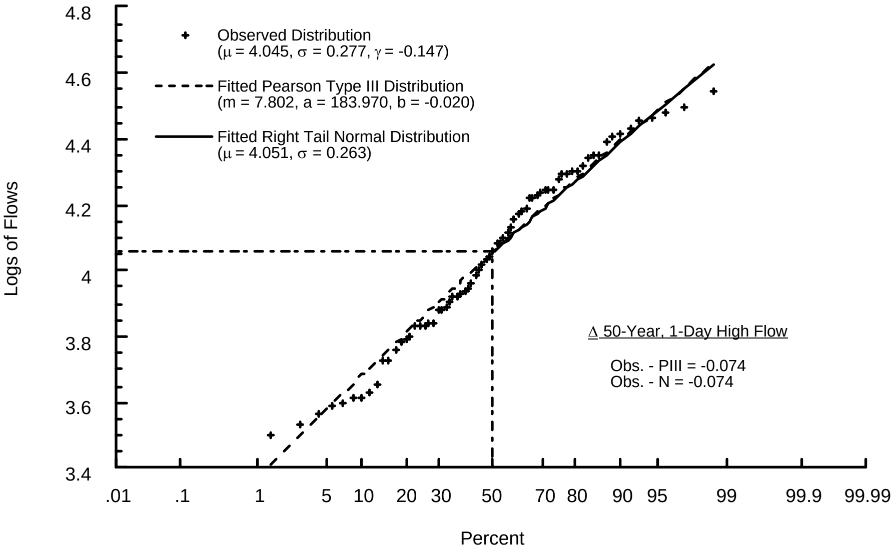


Figure I-4: Upper Mississippi Basin (Maquaketa at Maquaketa, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

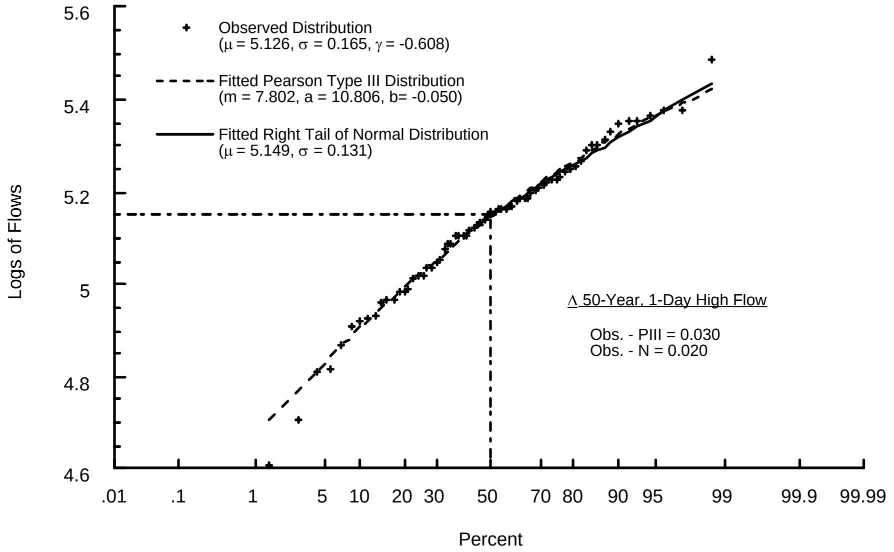


Figure I-5: Upper Mississippi Basin (Mississippi at Clinton, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

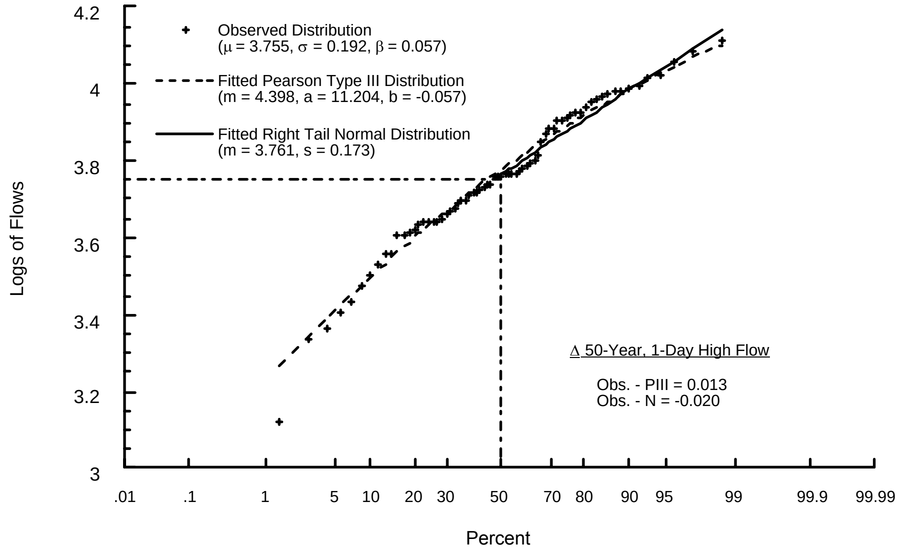


Figure I-6: Upper Mississippi Basin (Rock at Afton, WI) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

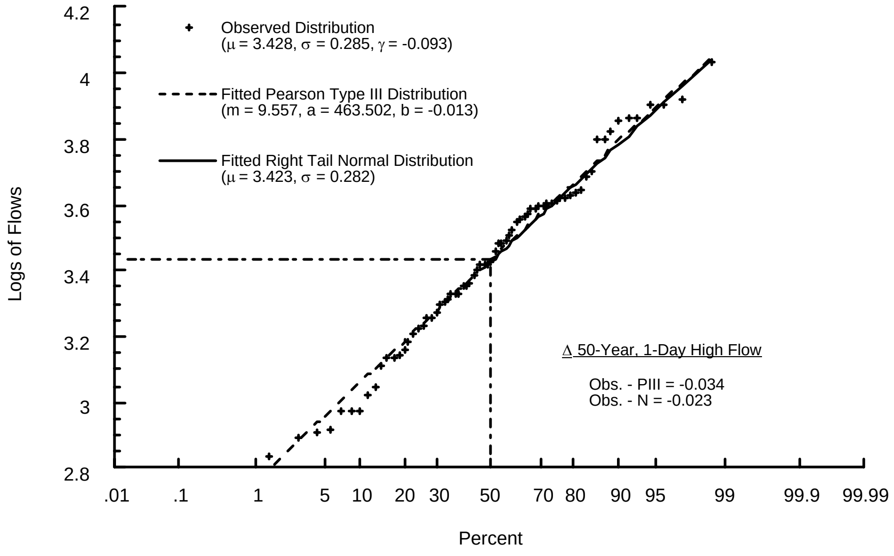


Figure I-7: Upper Mississippi Basin (Sugar at Broadhead, WI) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

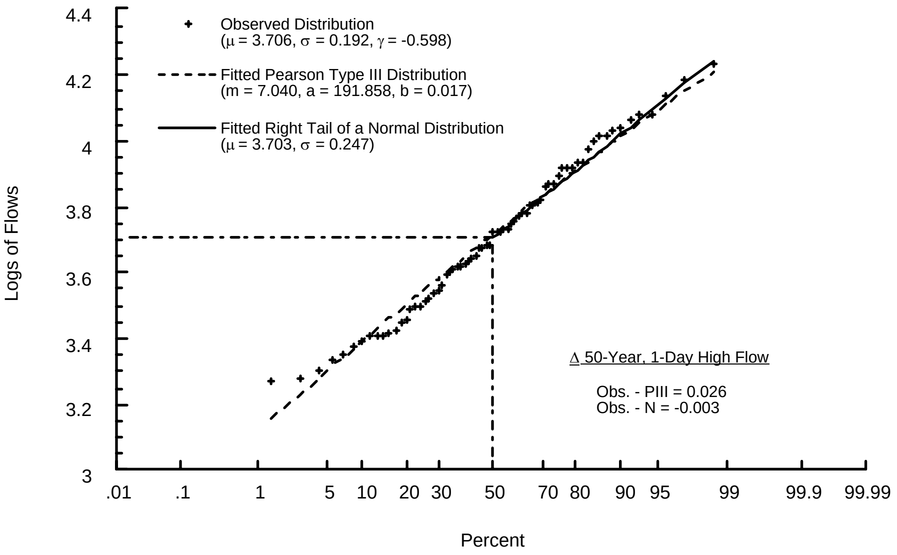


Figure I-8: Upper Mississippi Basin (Pecatonica at Freeport, IL) – Distribution of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

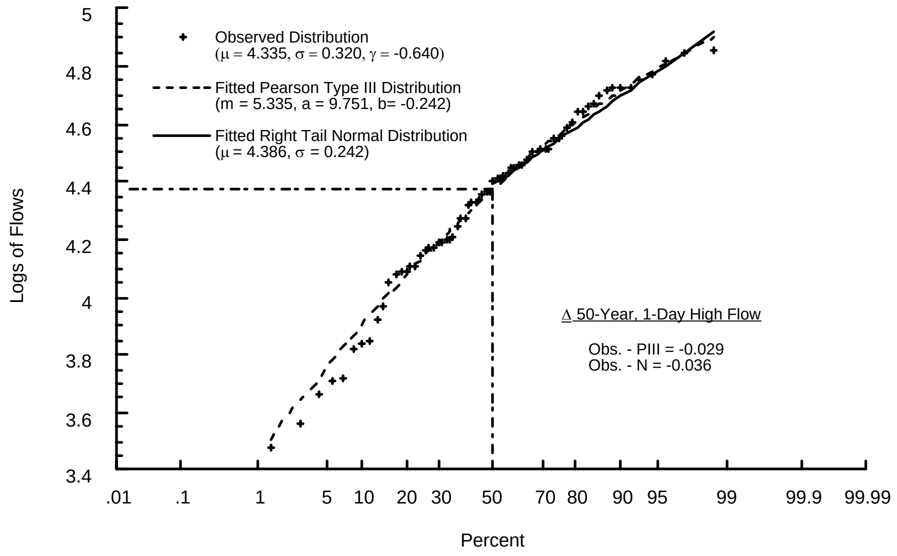


Figure I-9: Upper Mississippi Basin (Cedar at Cedar Rapids, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

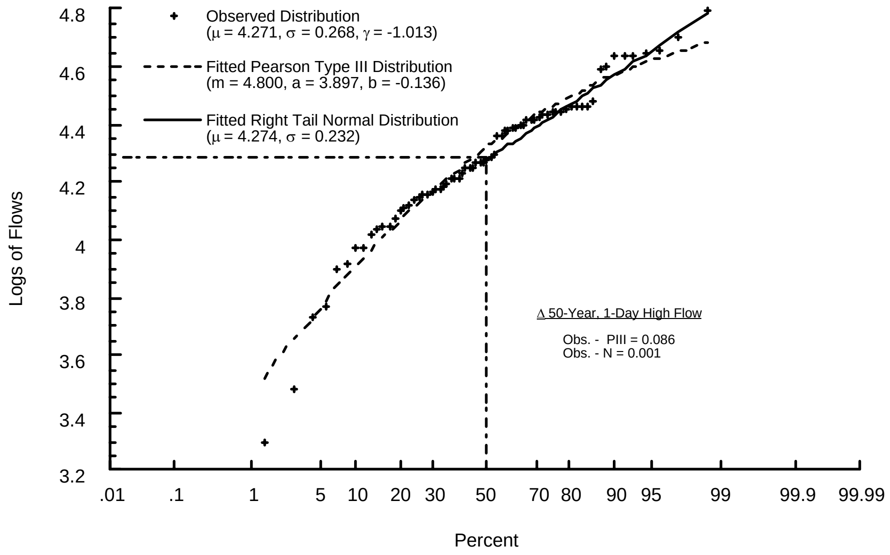


Figure I-10: Upper Mississipi Basin (Skunk at Augusta, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distrinution and with a Right-Tail Normal Distribution

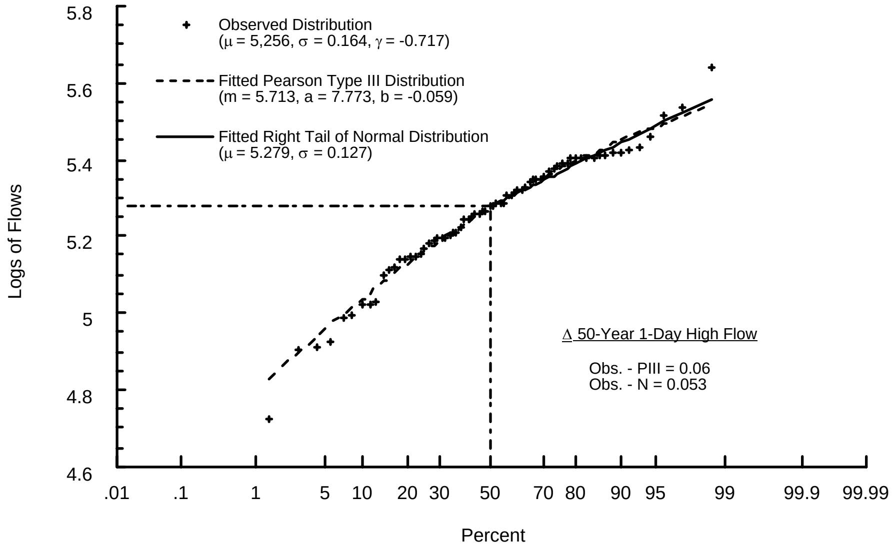


Figure I-11: Upper Mississippi Basin (Mississippi at Keokuk, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution.

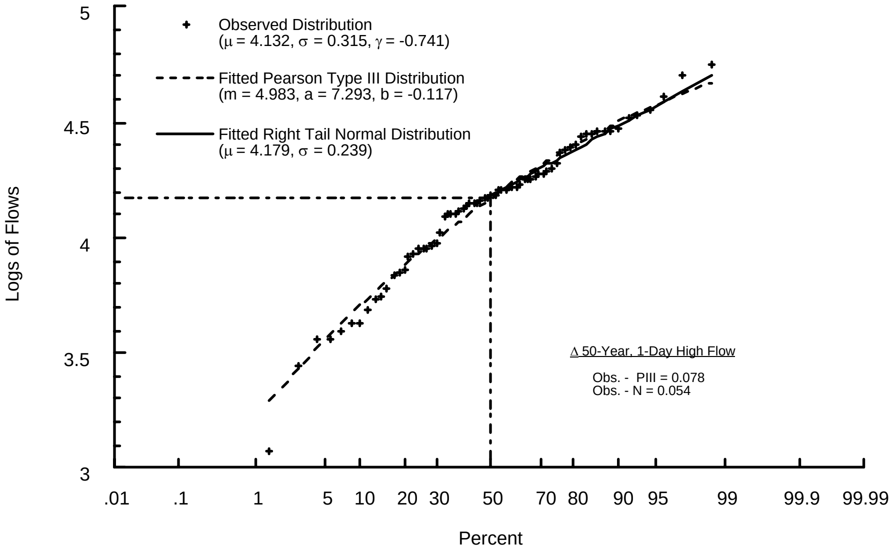


Figure I-12: Upper Mississippi Basin (Des Moines at Stratford, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

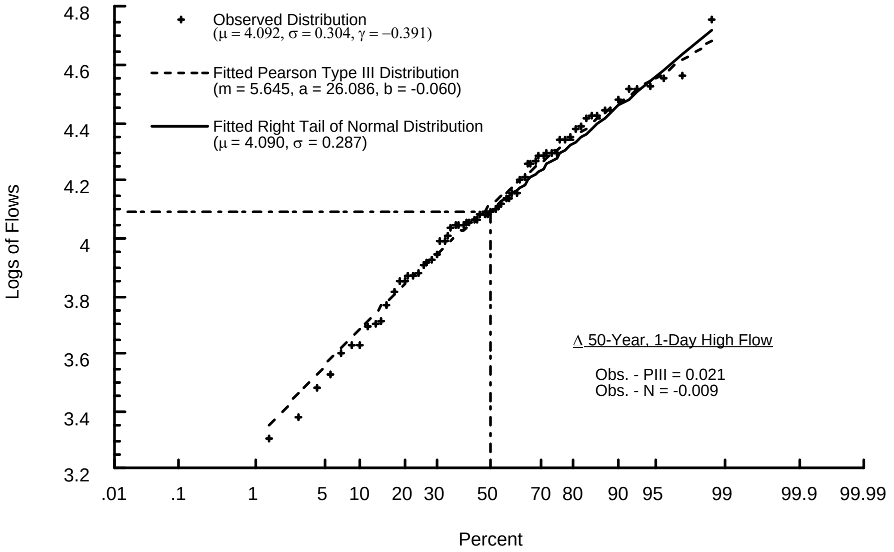


Figure I-13: Upper Mississippi Basin (Raccoon at Van Meter, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

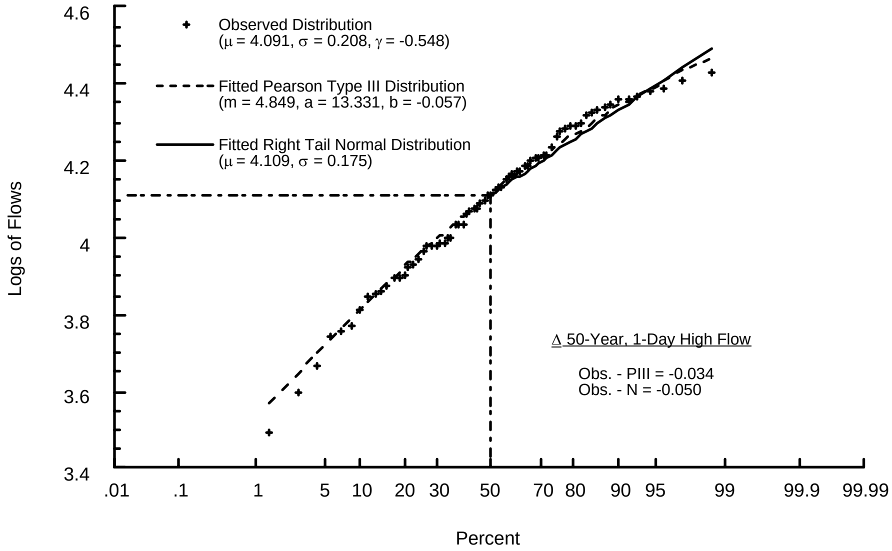


Figure I-14: Upper Mississippi Basin (Iroquois at Chebanse, IL) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

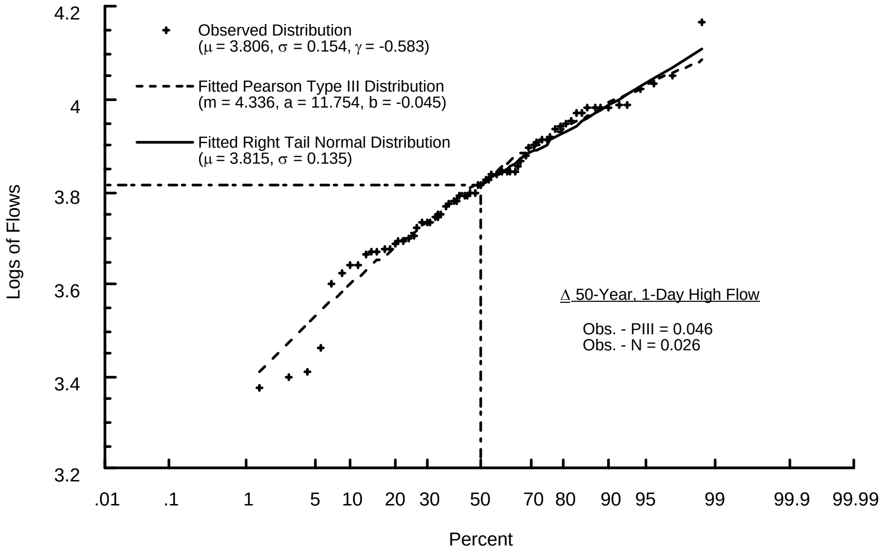


Figure I-15: Upper Mississippi Basin (Kankakee at Momence, IL) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

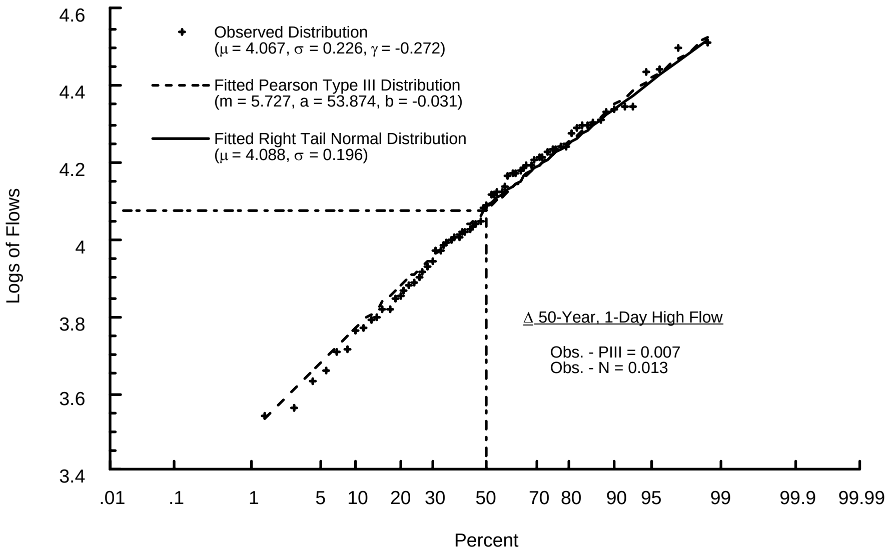


Figure I-16: Upper Mississippi Basin (Spoon at Seville, IL) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson TYpe III DIstribution and with a Right-Tail Normal Distribution

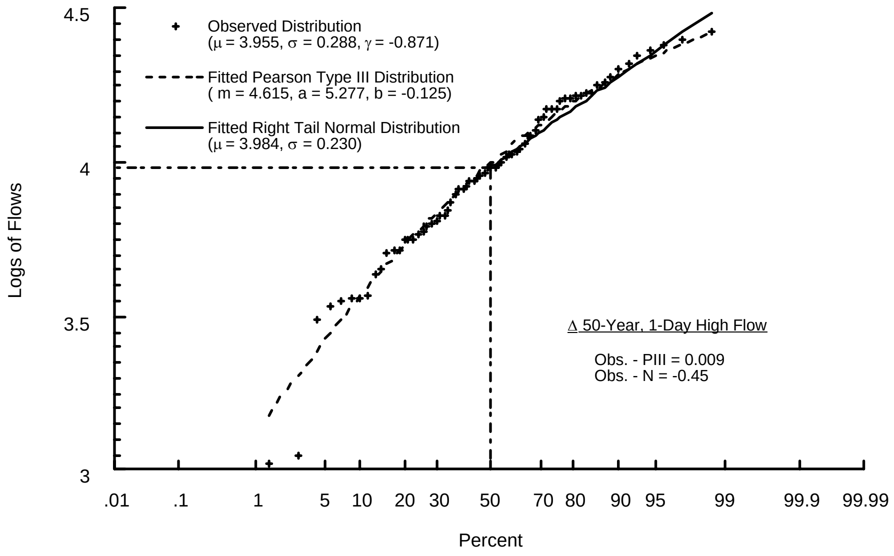


Figure I-17: Upper Mississippi Basin (La Moines at Ripely, IL) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

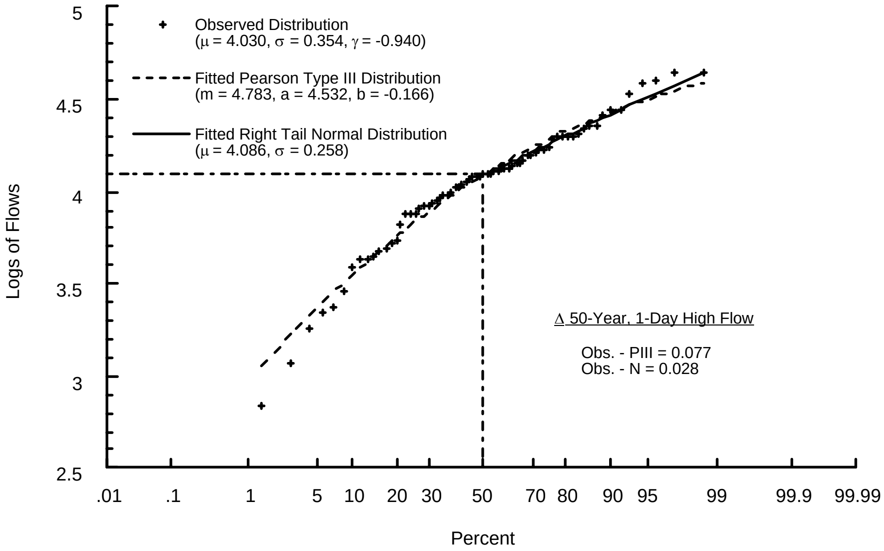


Figure I-18: Upper Mississippi Basin (Meramec at Steelville, MO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

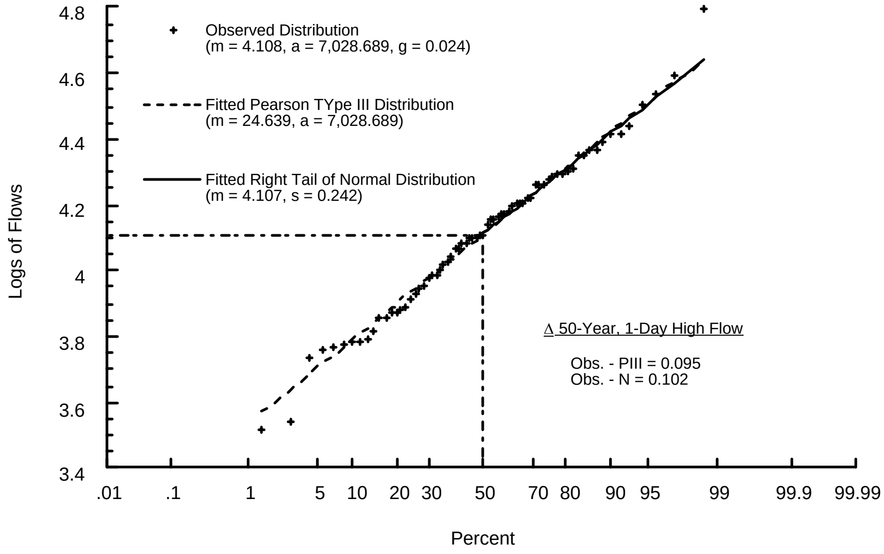


Figure I-19: Upper Mississippi Basin (Bourbeuse at Union, MO) – Distribution in Log Space of annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

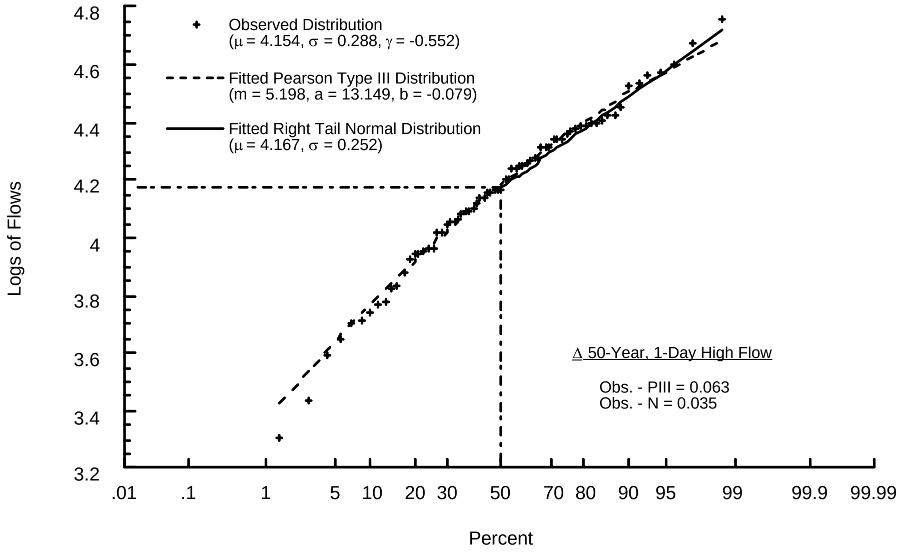


Figure I-20: Upper Mississippi Basin (Big at Byrnesville, MO) - Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

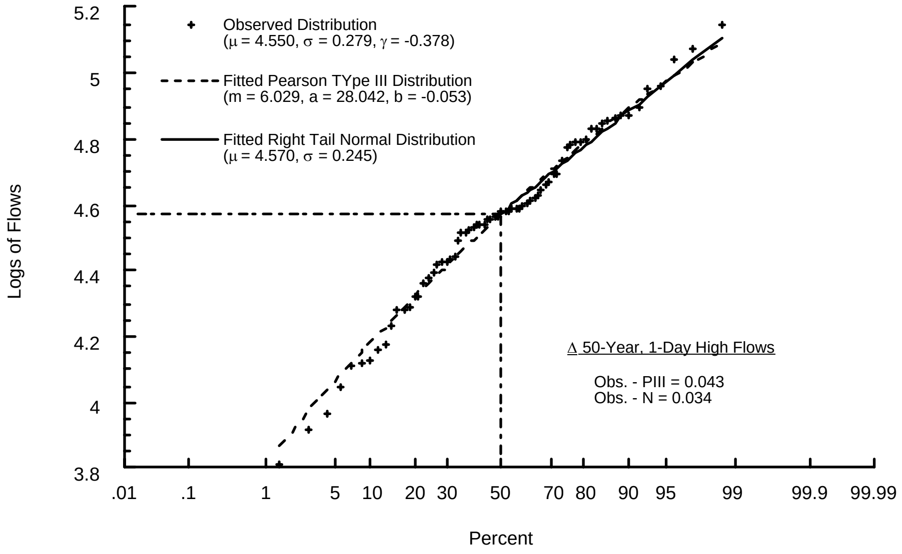


Figure I-21: Upper Mississippi Basin (Meramec at Eureka, MO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III distribution and with a Right -Tail Normal ditribution

Missouri Basin

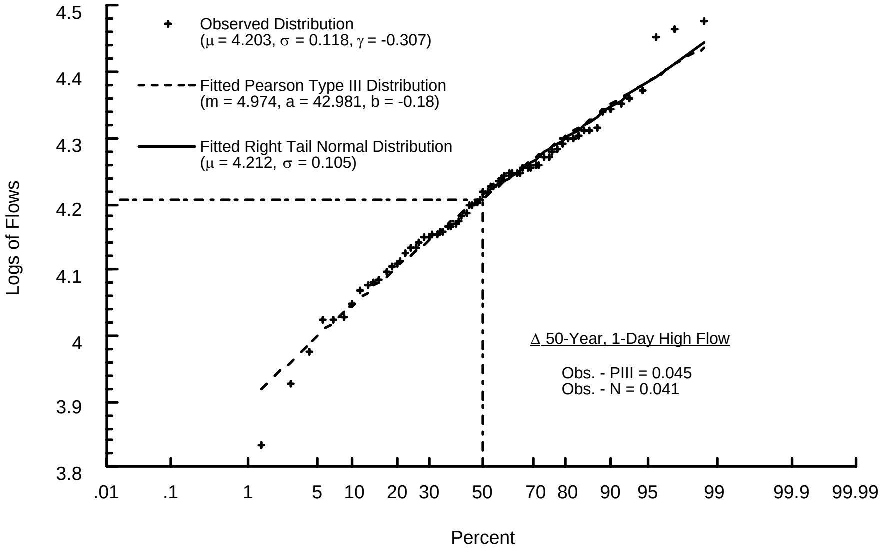


Figure I-22: Missouri Basin (Yellowstone at Augusta, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

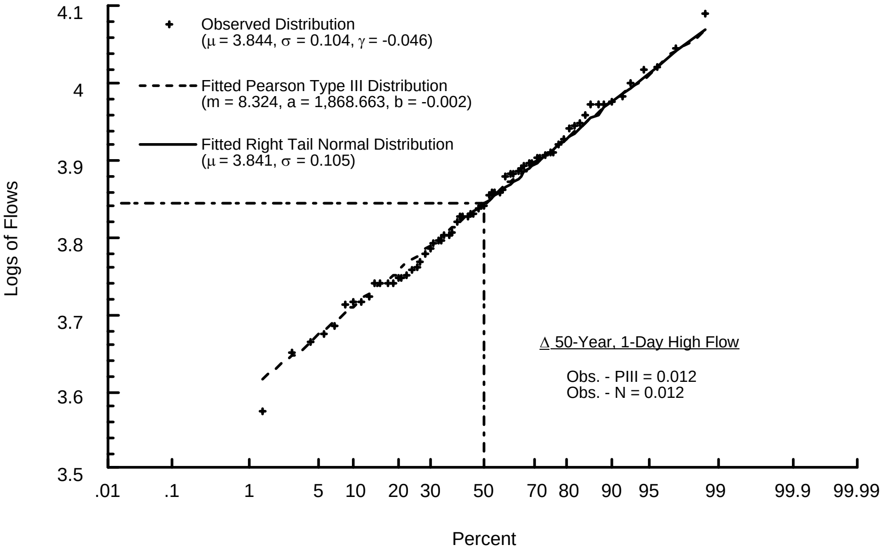


Figure I-23: Missouri Basin (Clarks Fork at Belfry, MT) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

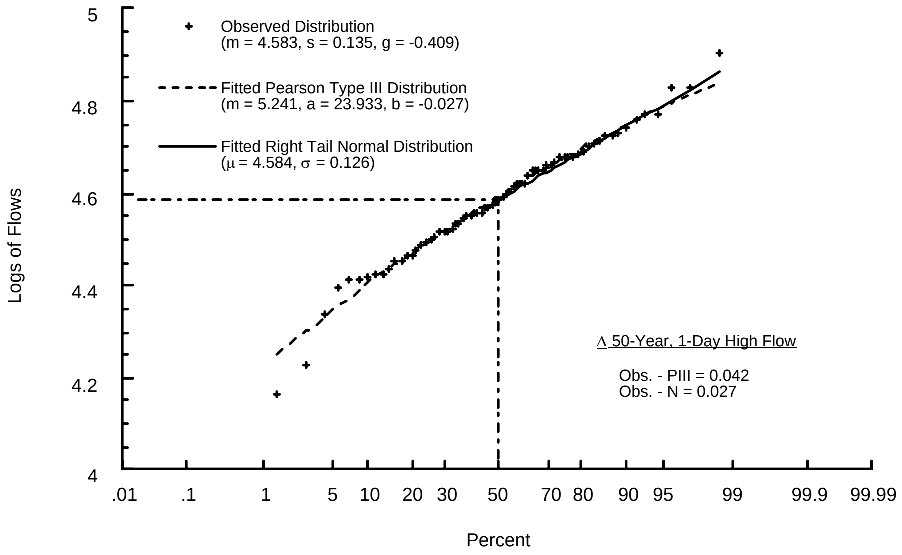


Figure I-24: Missouri Basin (Yellowstone at Billings, MT) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

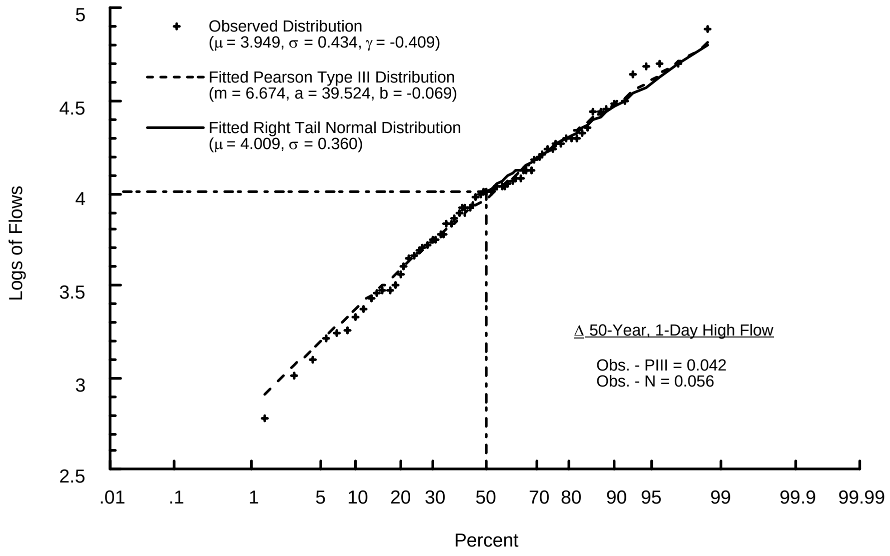


Figure I-25: Missouri Basin (Big Sioux at Akron, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

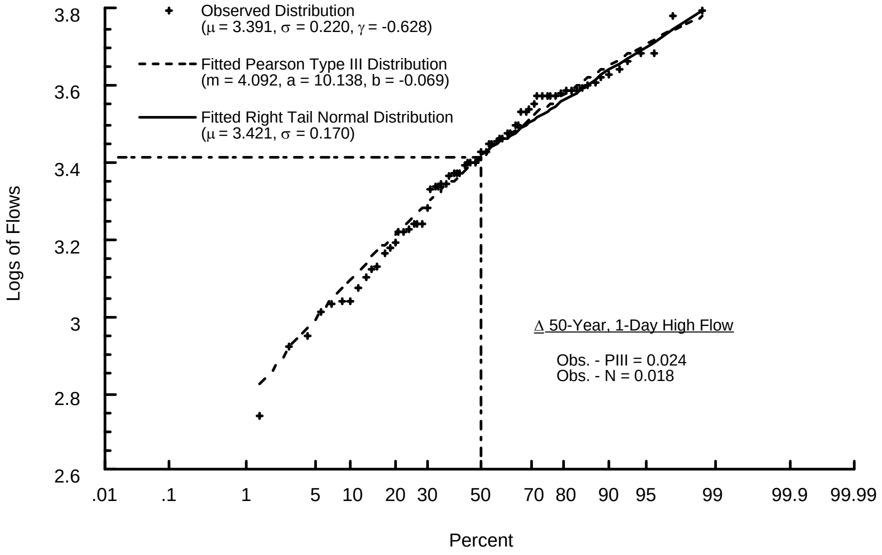


Figure I-26: Missouri Basin (North Platte at Northgate, CO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

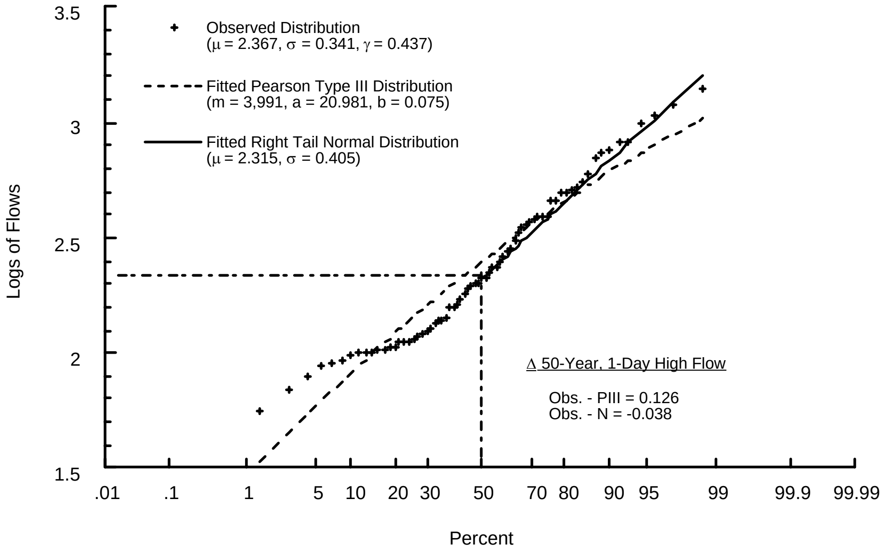


Figure I-27: Missouri Basin (Bear at Morrison, CO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson TYpe III Distribution and with a Right-Tail Normal Distribution

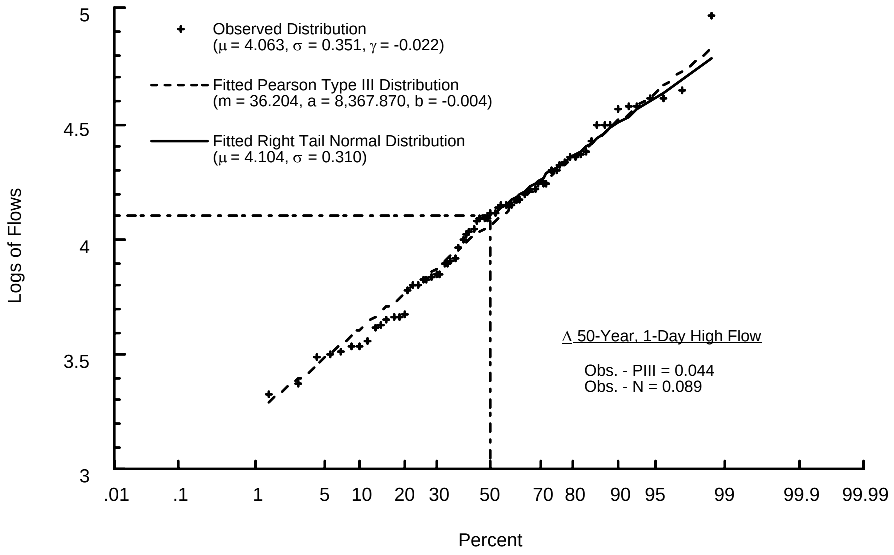


Figure I-28: Missouri Basin (Elkhorn at Waterloo, NE) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson TYpe III Distribution and with a Right-Tail Normal Distribution

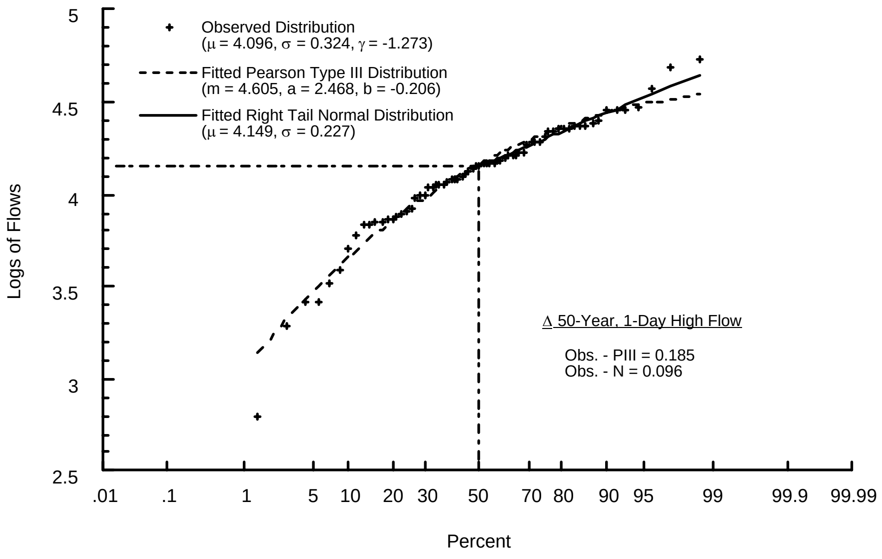


Figure I-29: Missouri Basin (Nishnabottna at Hamburg, IA) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Righ-Tail Normal Distribution

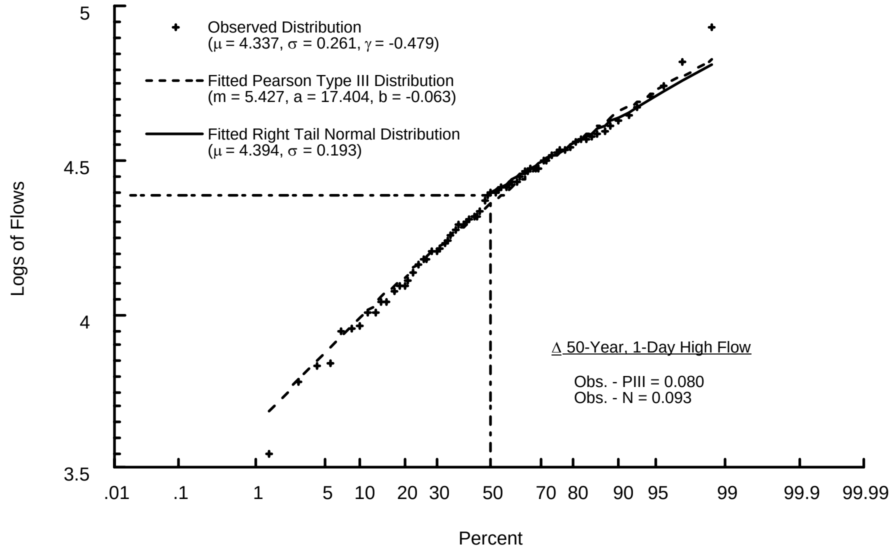


Figure I-30: Missouri Basin (Grand at Gallatin, MO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson TYpe III Distribution and witha Right-Tail Normal Distribution

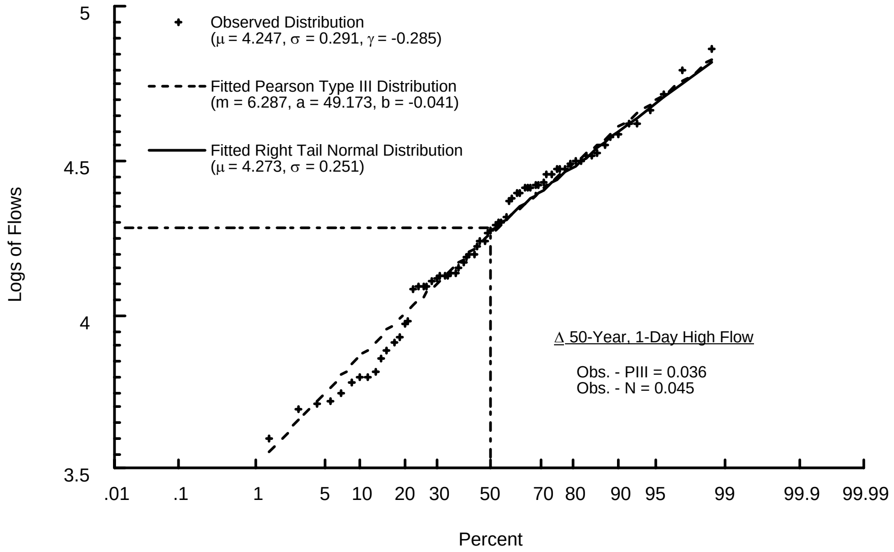


Figure I-31: Missouri Basin (Thompson at Trenton, MO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

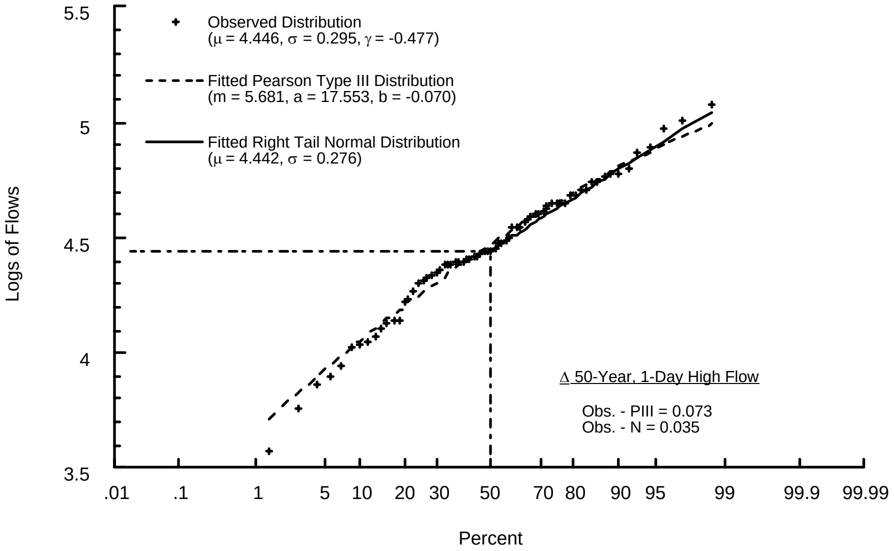


Figure I-32: Missouri Basin (Gasconade at Jerome, MO) – Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution

Appendix J: Regional Distributions of Sequences of Annual 1-Day High Flows in Log Space and Quasi-Log Space Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

Log Space

The regional distributions in log space or quasi-log space are fitted with Right-Tail Normal distributions using each of the three methods. Only the right tails of the Right-Tail Normal distributions are shown. Both the left and the right tail of the Pearson Type III distributions are shown.

Upper Mississippi Basin

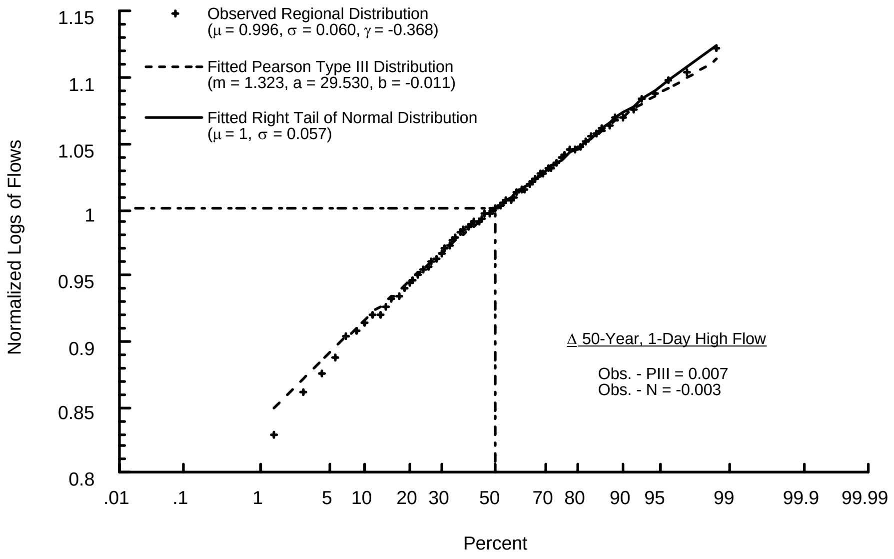


Figure J-1: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Righ-Tail Normal Distribution – Inflection Point Method

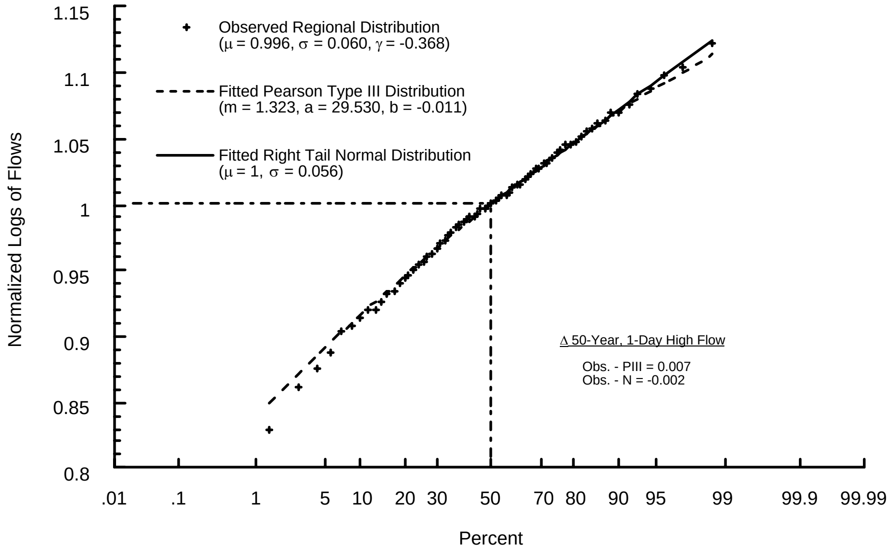


Figure J-2: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III distribution and with a Right-Tail Normal Distribution – 35-Point Method

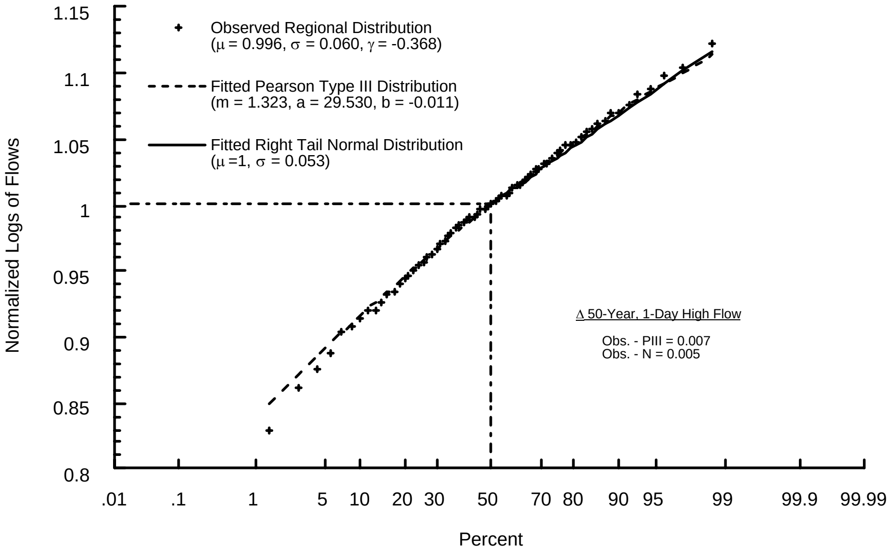


Figure J-3: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution – Mirrored-Spread

Missouri Basin

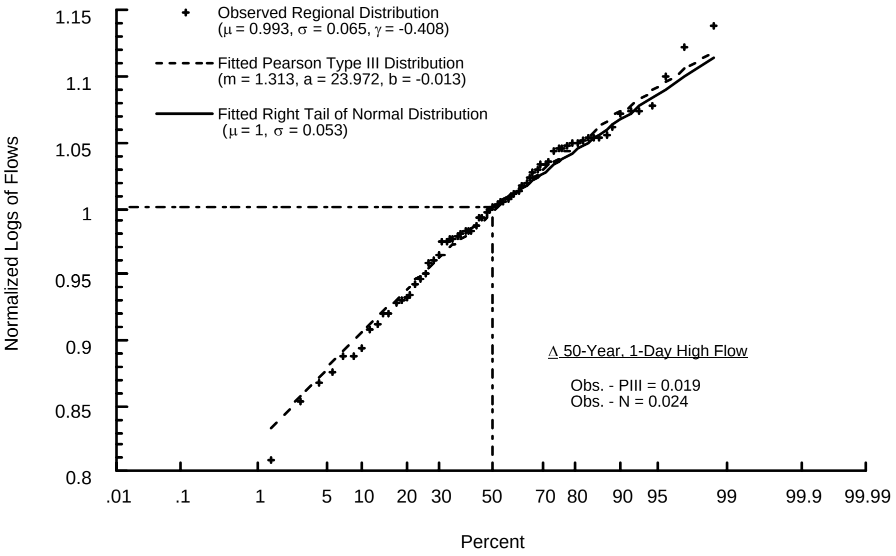


Figure J-4: Missouri Basin – Regionalized Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III distribution and with a Right-Tail Normal Distribution – Inflection Point Method

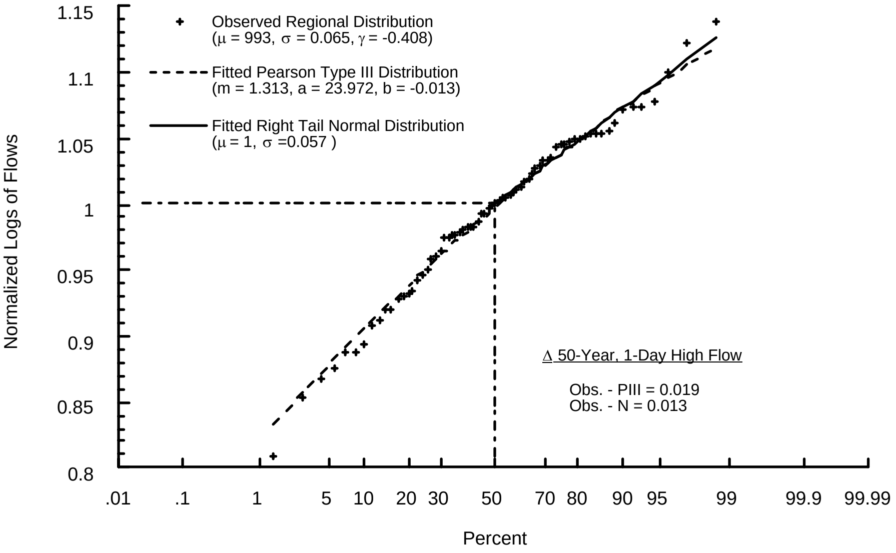


Figure J-5: Missouri Basin – Regionalized Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III distribution and with a Right-Tail Normal Distribution – 35-Point Method

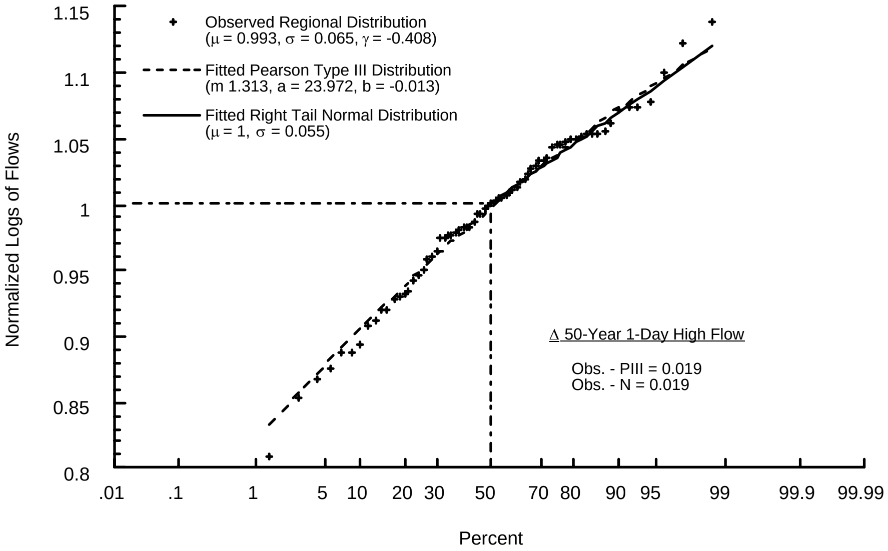


Figure J-6: Missouri Basin – Regionalized Distribution in Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution – Mirrored-Spread Method

Quasi-Log Space

Upper Mississippi Basin

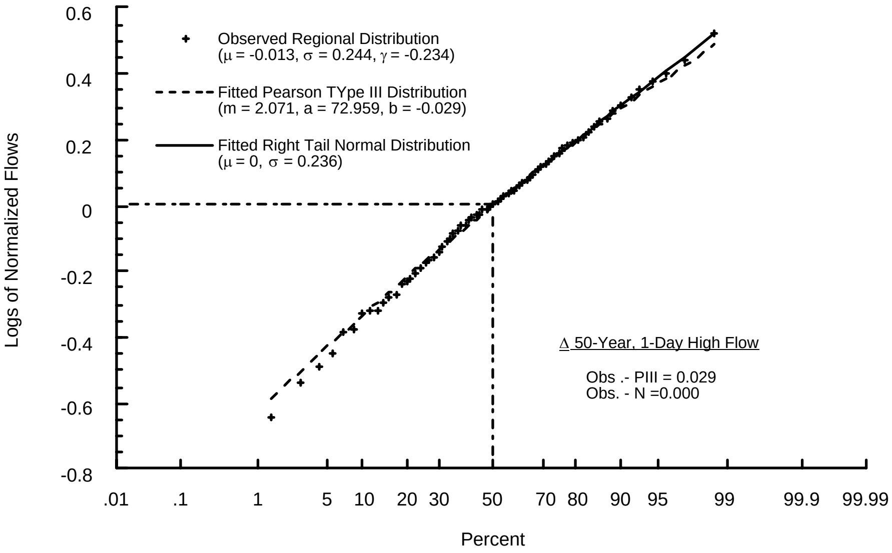


Figure J-7: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution – Inflection Point Method

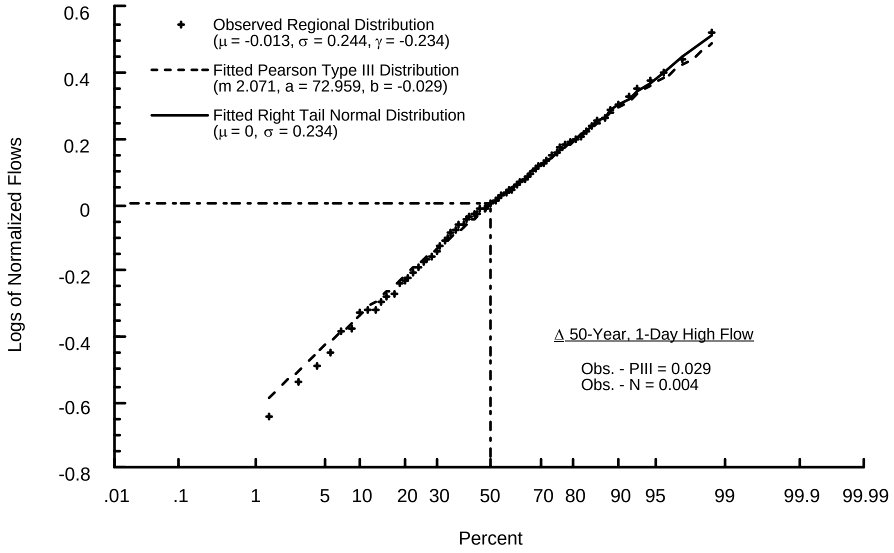


Figure J-8: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution – 35-Point Method

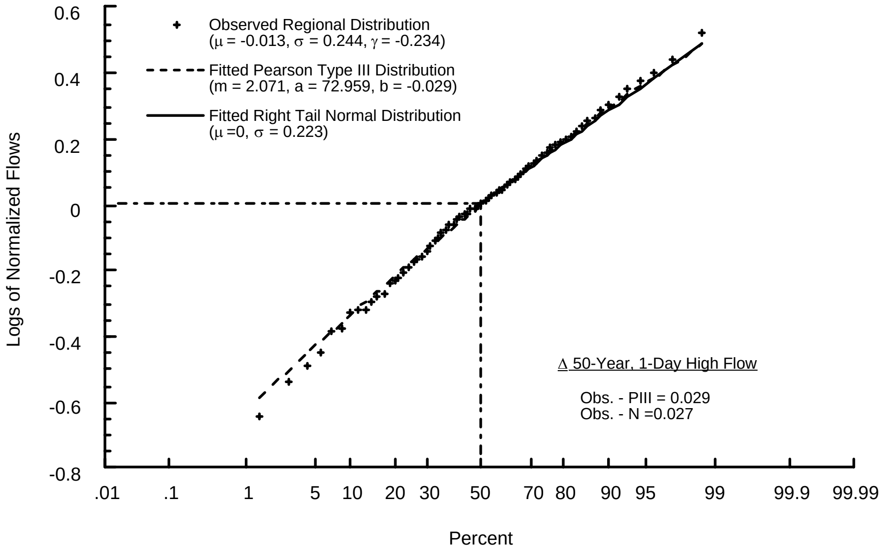


Figure J-9: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail of a Normal Distribution – Mirrored-Spread Method

Missouri Basin

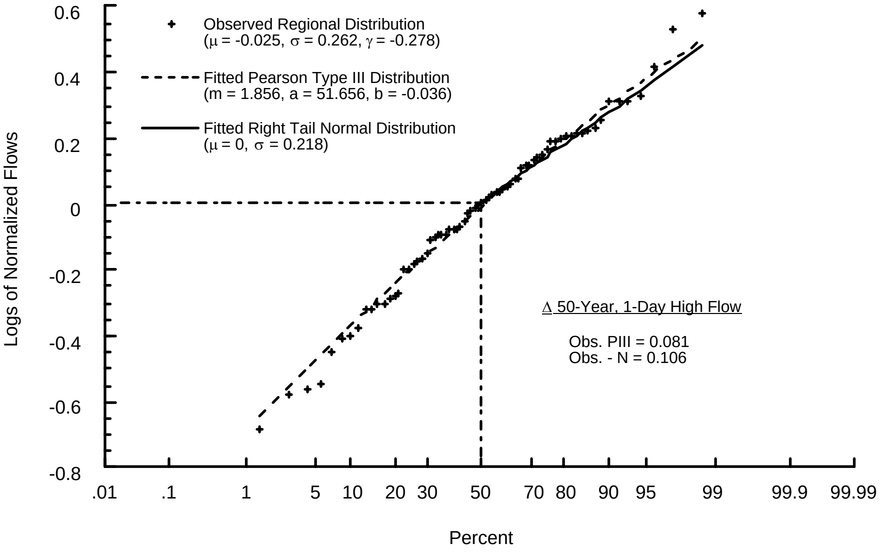


Figure J-10: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution- Inflection Point Method

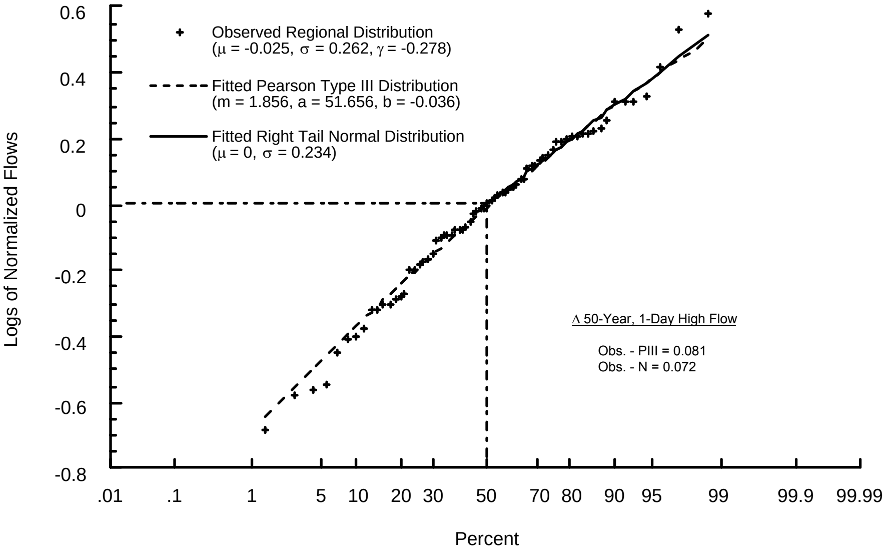


Figure J-11: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution – 35-Point Method

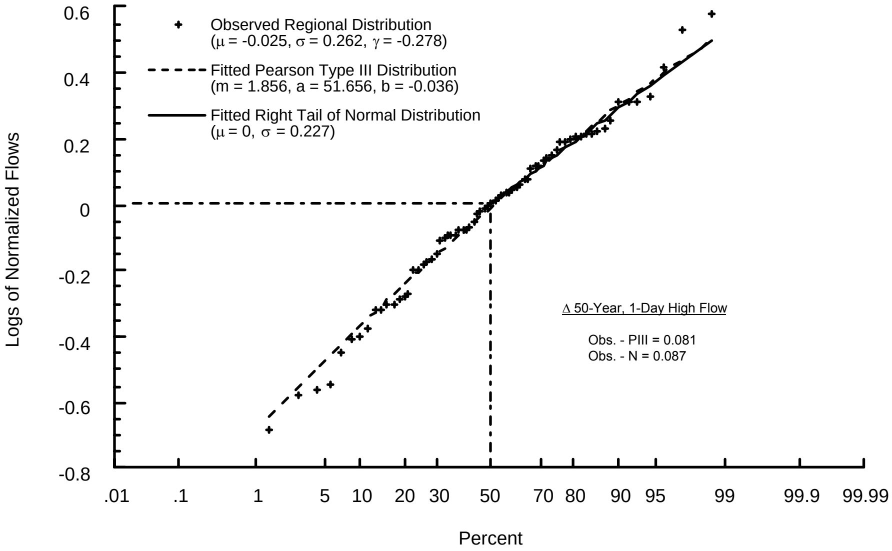


Figure J-12: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual 1-Day High Flows Fitted with a Pearson Type III Distribution and with a Right-Tail Normal Distribution – Mirrored-Spread

Appendix K: At-Site Distributions of Sequences of Annual Peak Flows Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

For consistency with the fitting of Right-Tail Normal distributions to at-site sequences of annual 1-day high flows (Appendices I), the at-site sequences of annual peak flows are also fitted with Right-Tail Normal distributions using the mirrored spread method. In doing so, the Pearson Type III distribution provides better fits for most of the Missouri basin sequences than the Right-Tail Normal distributions. If other methods are used, then the Right-Tail Normal distributions would provide better fits for most of the sites than the Pearson Type III distribution.

Upper Mississippi Basin

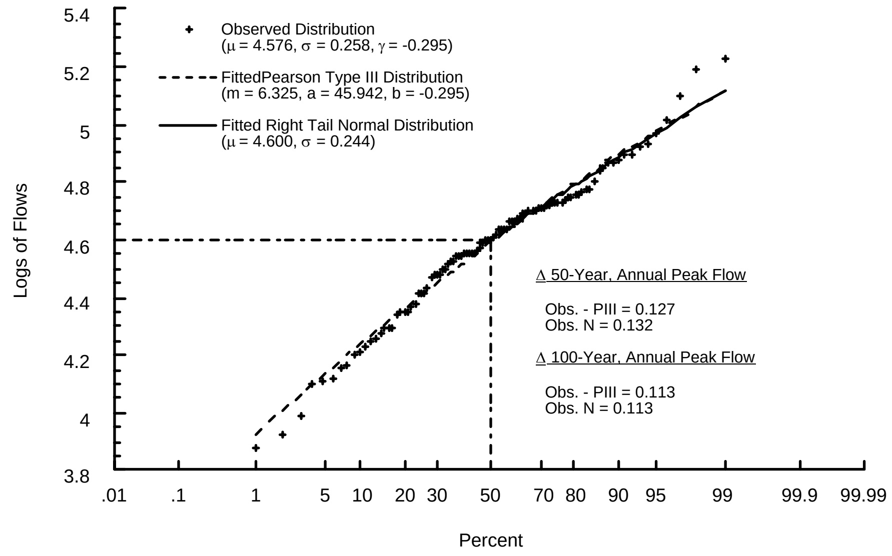


Figure K-1: Upper Mississippi Basin (St. Paul, MN) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

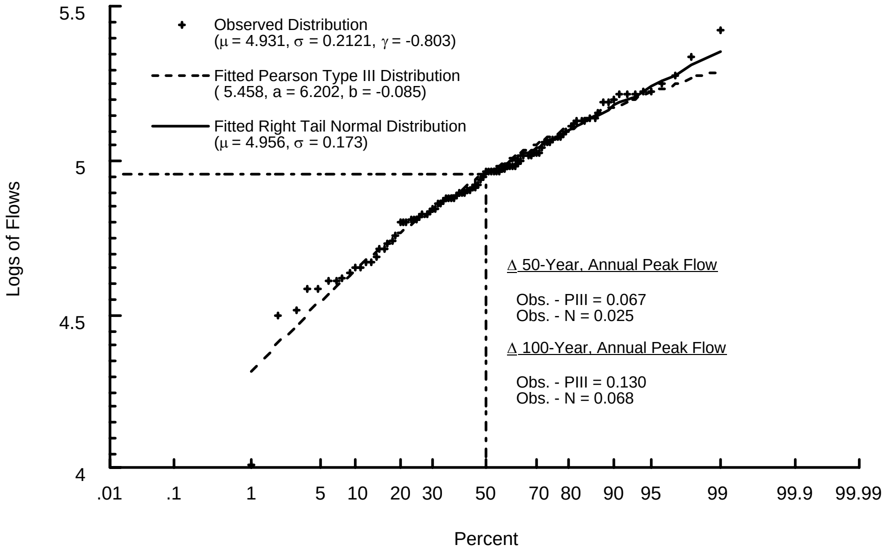


Figure K-2: Upper Mississippi Basin (Winona, MN) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

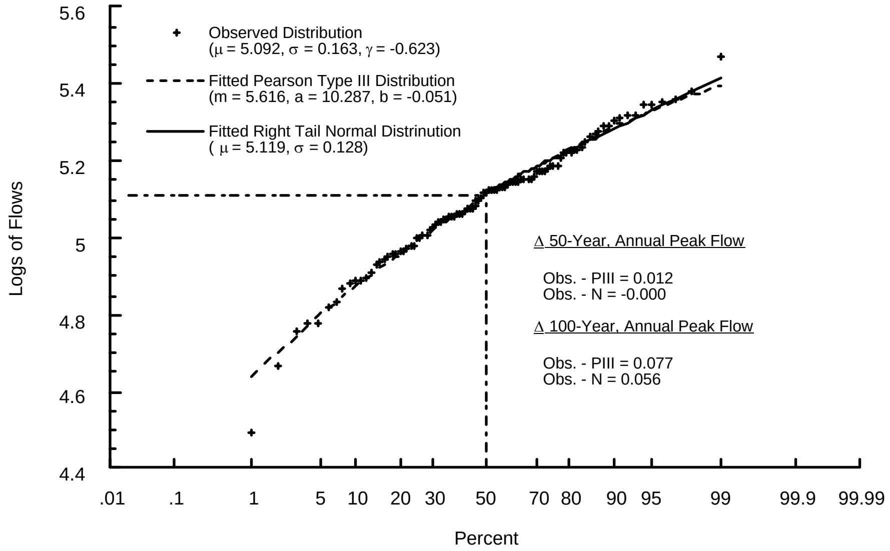


Figure K-3: Upper Mississippi Basin (Dubuque, IA) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

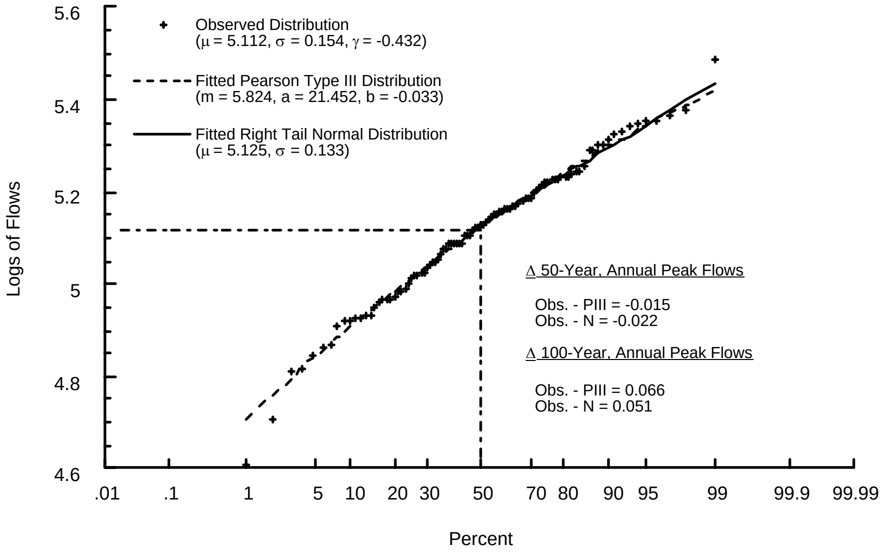


Figure K-4: Upper Mississippi Basin (Clinton, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III distribution and with a Right Tail Normal Distribution

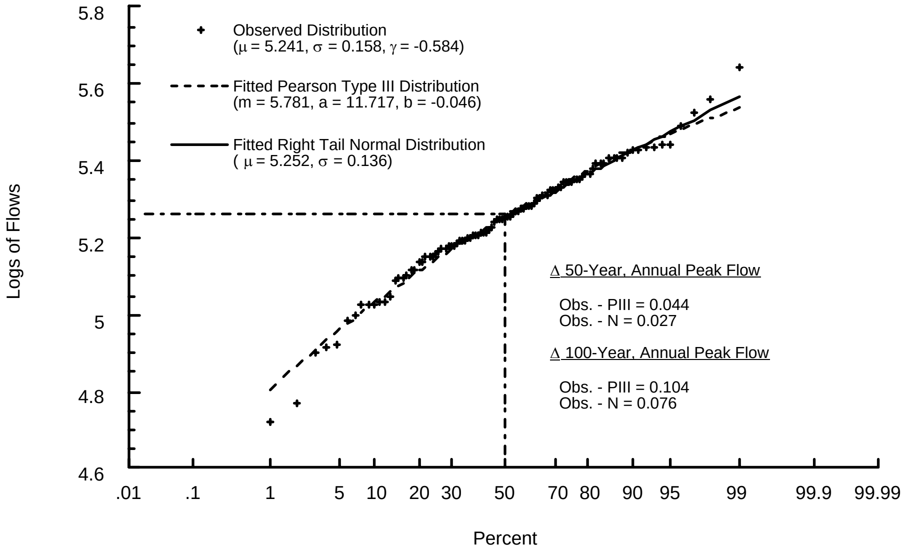


Figure K-5: Upper Mississippi Basin (Keokuk, IA) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

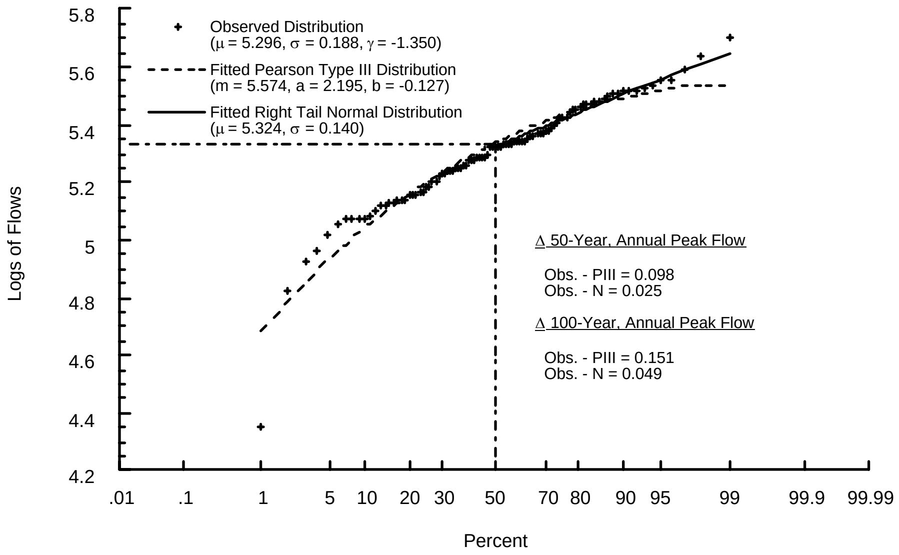


Figure K-6: Upper Mississippi Basin (Hannibal, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

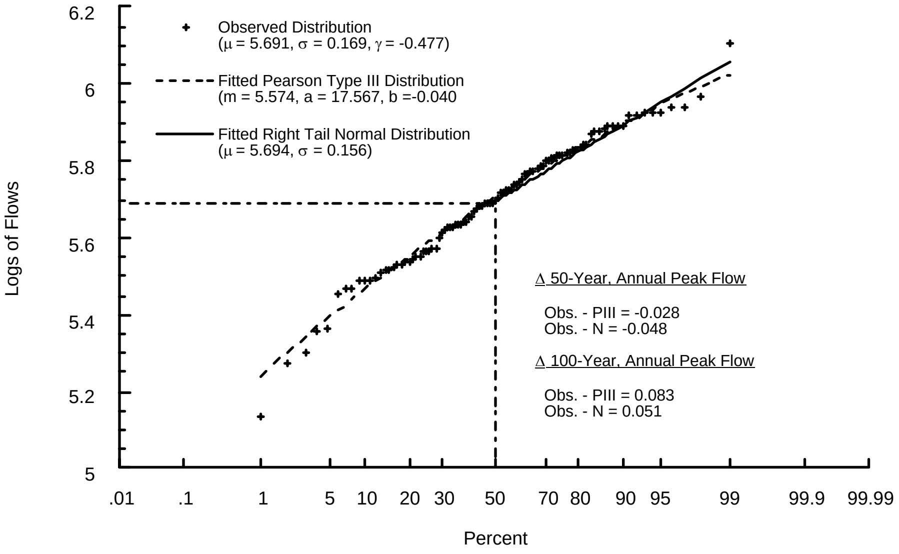


Figure K-7: Upper Mississippi Basin (St. Louis, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

Missouri Basin

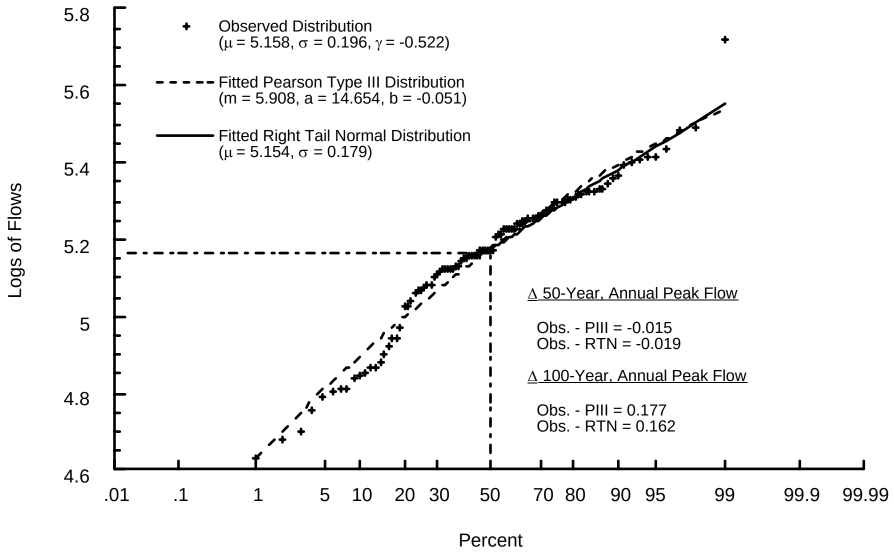


Figure K-8: Missouri Basin (Sioux City, IA) _ Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

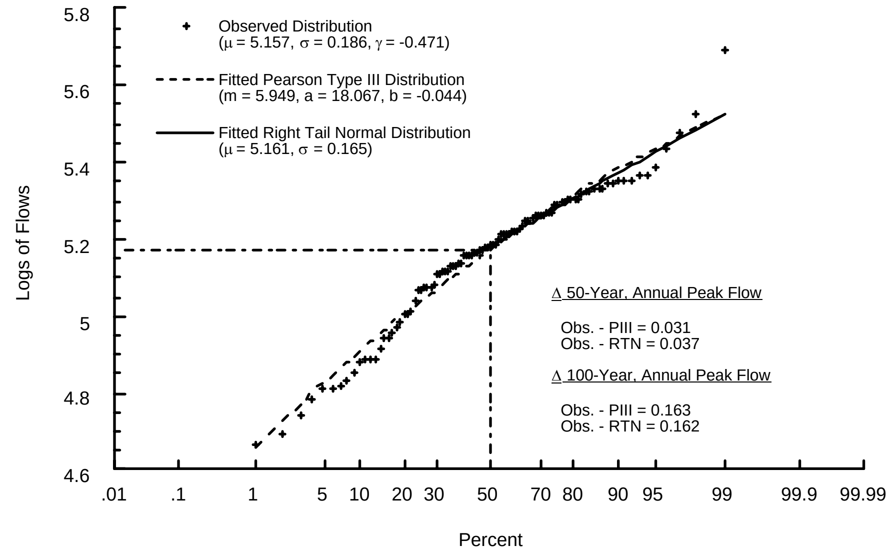


Figure K-9: Missouri Basin (Omaha, NE) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

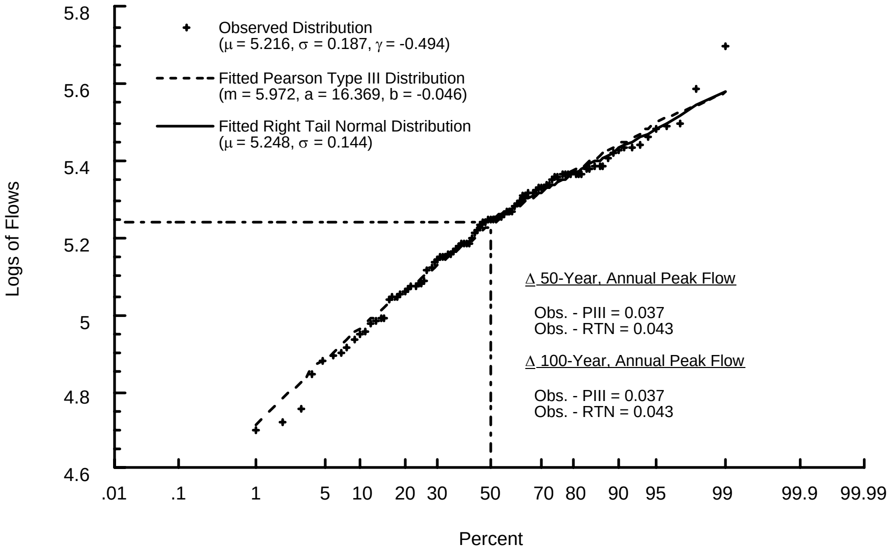


Figure K-10: Missouri Basin (Nebraska City, NE) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

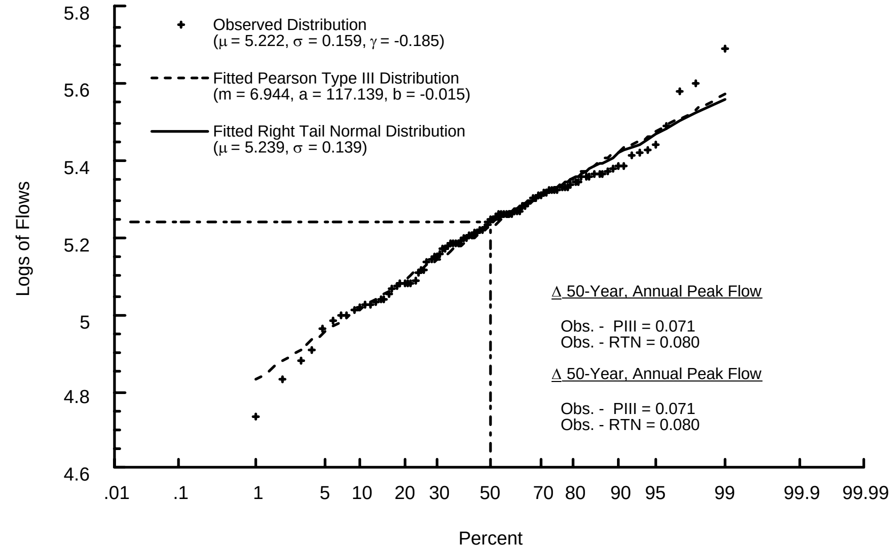


Figure K-11: Missouri Basin (St. Joseph, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

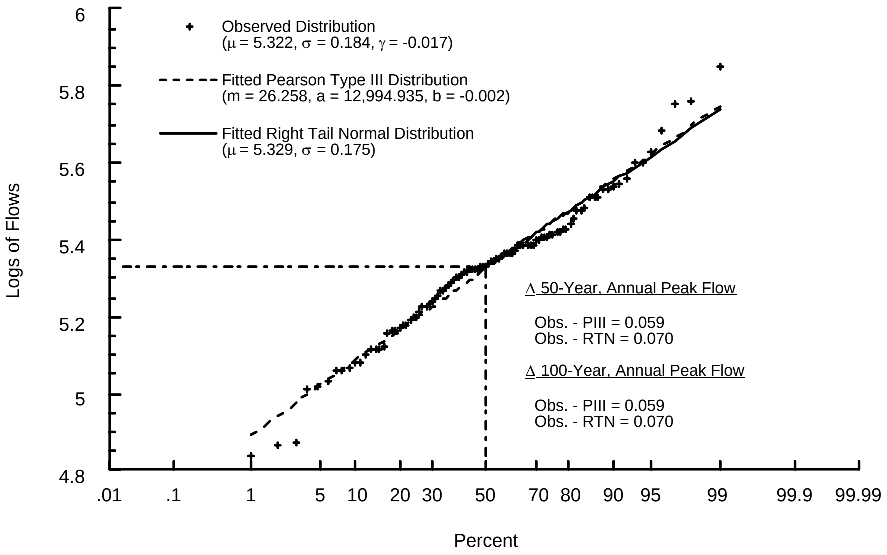


Figure K-12: Missouri Basin (Kansas City, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

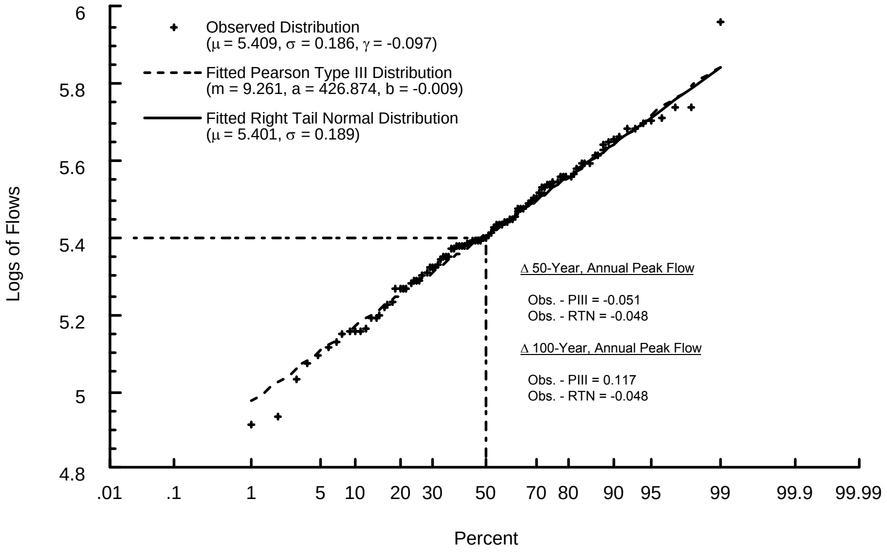


Figure K-13: Missouri Basin (Bonneville, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

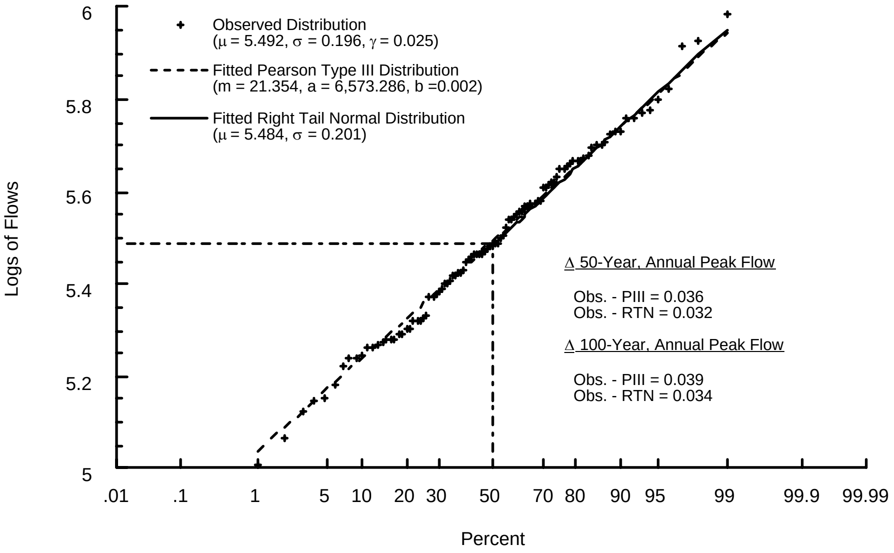


Figure K-14: Missouri Basin (Hermann, MO) – Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

Appendix L: Regional Distributions of Sequences of Annual Peak Flows in Log Space and Quasi-Log Space Fitted with a Pearson Type III Distribution and a Right-Tail Normal Distribution

The regional distributions in log space or quasi-log space are fitted with Right-Tail Normal distributions using each of the three methods. Only the right tails of the Right-Tail Normal distributions are shown. Both the left and the right tail of the Pearson Type III distributions are shown.

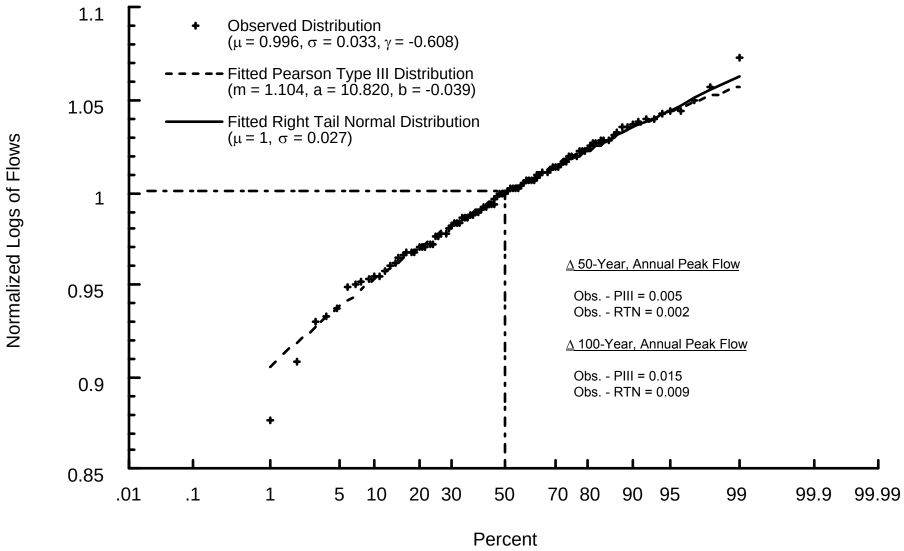


Figure L-1: Upper Mississippi Basin – Regionalized Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

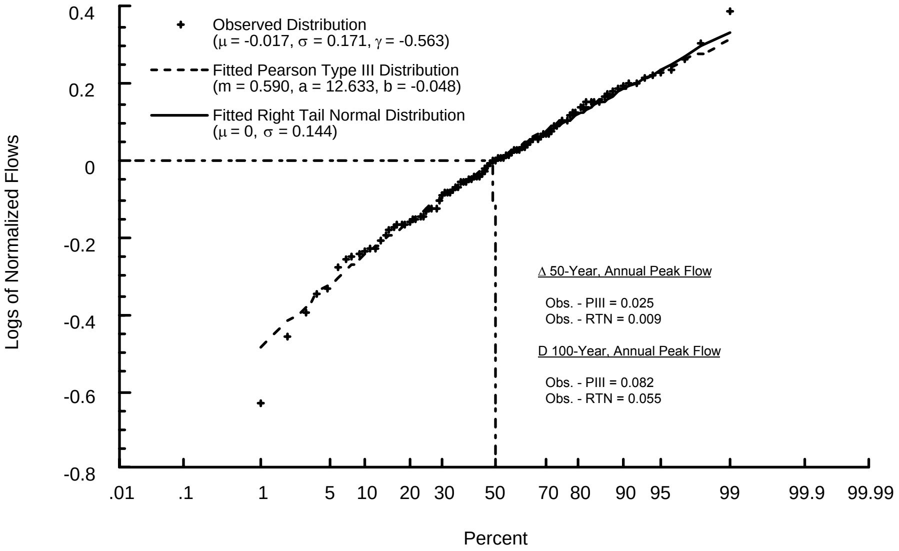


Figure L-2: Upper Mississippi Basin – Regionalized Distribution in Quasi-Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

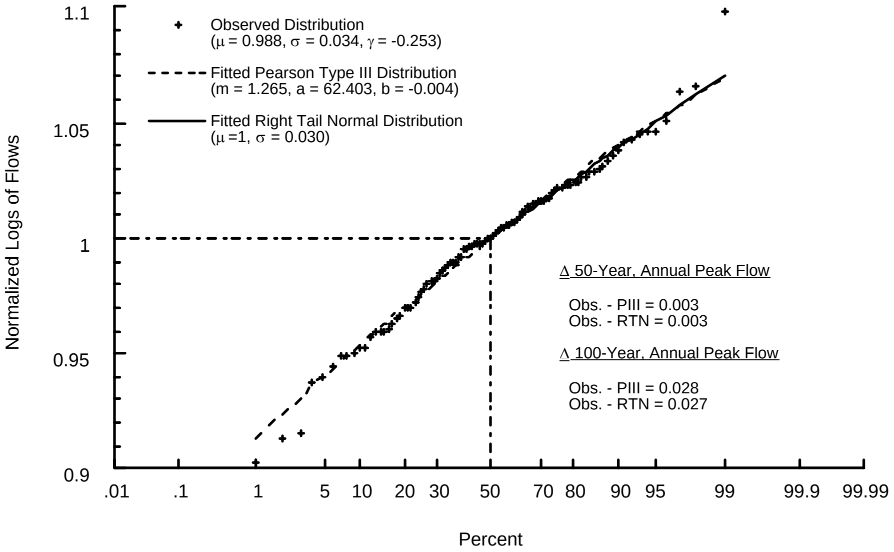
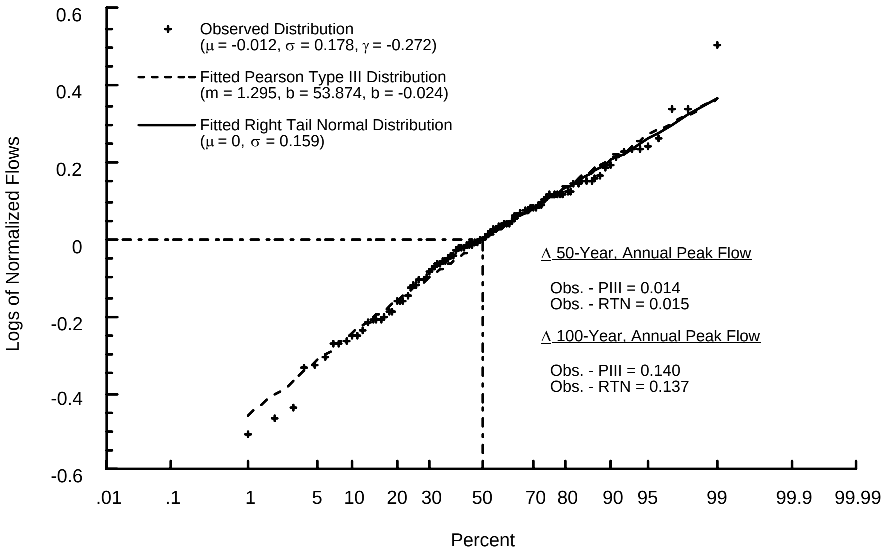


Figure L-3: Missouri Basin – Regionalized Distribution in Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution



Δ 50-Year, Annual Peak Flow

Obs. - PIII = 0.014
Obs. - RTN = 0.015

Δ 100-Year, Annual Peak Flow

Obs. - PIII = 0.140
Obs. - RTN = 0.137

Figure L-4: Missouri Basin – Regionalized Distribution in Quasi-Log Space of Annual Peak Flows Fitted with a Pearson Type III Distribution and with a Right Tail Normal Distribution

Appendix M: Bivariate Distributions with Specified Marginal Distributions

In a given hydrologic region, flow sequences are to a varying degree correlated, where correlation is a measure of linear dependence. Information extracted from the sequences on a sequence by sequence basis is to some degree redundant. The degree of redundancy varies directly with the degree of correlation. At one extreme where the sequences are perfectly correlated with one another, the degree of redundancy equals one, implying that there is effectively one sequence in the region. At the other extreme, where the sequences are uncorrelated with one another, the degree of redundancy is equal to the number of sequences in the region. In general, the correlation between the sequences lies between the two extremes. All other things being equal, the greater the distance between the locations of the gaged sites in the region, the smaller is the correlation between the sequences.

In assessing information extracted from the sequences collectively, account must be taken of the correlation structure among the sequences. Generally the correlation structure is interpreted as deriving from random variables distributed as multivariate Normal or as multivariate Log-Normal. Because flow sequences exhibit values of skewness that can not be statistically accepted as arising by chance, the multivariate Normal interpretation of the correlation structure among the sequences is questionable, though it might be accepted in order to arrive at a first order assessment of the extracted information. A more acceptable assessment of the information is that based on interpreting the correlation structure of the logs of the flows as deriving from random variables distributed as multivariate Normal.

In dealing with flood flows, hydrologic practice within federal agencies is guided by Bulletin 17-B which requires the agencies to use the Log-Pearson Type III distribution unless sufficient reason can be given to the use of another distribution. If Bulletin 17-B were to be extended to the multivariate case, then the correlation structure among the flood sequences in a region would be interpreted as deriving from random variables whose multivariate distribution is Log-Pearson Type III. The correlation structure among the sequences of the logs of the flows would be interpreted as arising from variables whose multivariate distribution is Pearson Type III.

To initiate study of the multivariate Log-Pearson Type III in hydrology, a procedure for generating bivariate Pearson Type III sequences is assessed. The procedure, introduced by Johnson (1978), is a general procedure for generating sequences relating to a bivariate distribution with specified marginal distributions. The marginal distributions can not be arbitrarily chosen. The distributions must be such that the random variables to which they relate are such that the distribution of the weighted sum of the variables is the same, apart from parameter values, as the distributions of the variables themselves conditioned on the random variables being independent and identically distributed.

The values of the sequences generated by Johnson's procedure may be exponentiated to derive sequences of Log-Pearson Type III bivariate sequences. Regional assessments, e.g. assessments of trend, persistence and flood risk, may be made in terms of bivariate Log-Pearson Type III sequences generated by Johnson's procedure. That assessment may be compared with that made using Log-Normal sequences generated by Johnson's procedure. The Log-Normal sequences generated by Johnson's procedure may be compared with the sequences derived through exponentiation of the standard bivariate Normal distribution.

Weighted Linear Combination

A simple procedure for generating bivariate sequences distributed with specified marginals and specified measure of dependence has been suggested by Johnson (1978). The procedures, referred to as the weighted linear combination is as follows. Let X and Y be two independent and identically distributed random variables. Define ξ and ζ as

$$\xi = X \tag{M-1}$$

$$\zeta = \beta X + (1-\beta)Y \tag{M-2}$$

where $0 \le \beta \le 1$ is the measure of dependence between ξ and ζ. Johnson (1978) notes that the procedure has two degrees of freedom, the measure of dependence, β, and the distribution of X identical to the distribution of Y, $F(X) \equiv F(Y)$.

Given that the distributions of X and Y are identical,

$$E\left[\xi^k\right] = E\left[X^k\right] = E\left[Y^k\right] \tag{M-3}$$

for $k \le k^* \ge 0$, where $k^* > 0$ is an integer. In order that the distributions of ξ and ζ are the same, apart from the values of their parameters, the distributions of X and Y can not be arbitrarily chosen.

Two distributions used extensively in hydrology are the Log-Normal and the Log-Pearson Type III. The distributions of X and Y cannot both be Log-Normal (Log-Pearson Type III) as the distribution of the sum of Log-Normal (Log-Pearson Type III) variables is not Log-Normal (Log-Pearson Pearson Type III). If the distributions of X and Y are both Normal (Pearson Type III), the distributions of ξ and ζ are both Normal (Pearson Type III), whereby the distributions of $exp(\xi)$ and $exp(\zeta)$ are both Log Normal (Pearson Type III). Through Eqs. (1) and (2), the bivariate structure having Log Normal marginals may be compared in a relatively straighforward manner with the bivariate structure having Log Pearson Type III marginals, where both bivariate structures characterized by the same degree of dependence. The dependence between the Log Normal (Log Pearson Type III) variables is weaker than that between the Normal (Pearson Type III) variables. Nonlinear transformation weakens the degree of dependence.

The means of X and Y are

$$\mu(X) = \mu(Y) \tag{M-4}$$

where

$$\mu(*) = E[*] \tag{M-5}$$

The variances of X and Y are

$$\sigma^2(X) = \sigma^2(Y) \tag{M-6}$$

where

$$\sigma^2(*) = E\left[*^2\right] - E^2[*] \tag{M-7}$$

The third central moments of X and Y are

$$\mu_3(X) = \mu_3(Y) \tag{M-8}$$

where

$$\mu_3(*) = E\left[*^3\right] - 3E\left[*^2\right]E[*] + 2E[*] \tag{M-9}$$

The fourth central moments of X and Y are

$$\mu_4(X) = \mu_4(Y) \tag{M-10}$$

where

$$\mu_4(*) = E\left[*^4\right] - 4E\left[*^3\right]E[*] + 6E\left[*^2\right]E^2[*] - 3E^4[*] \tag{M-11}$$

The coefficients of skewness of X and Y are

$$\gamma(X) = \gamma(Y) \tag{M-12}$$

where

$$\gamma(*) = \frac{\mu_3(*)}{\{\sigma^2(*)\}^{/2}} \qquad \text{(M-13)}$$

The coefficients of kurtosis of X and Y are

$$\lambda(X) = \lambda(Y) \qquad \text{(M-14)}$$

where

$$\lambda(*) = \frac{\mu_4(*)}{\{\sigma^2(*)\}^2} \qquad \text{(M-15)}$$

It follows that

$$E\left[\xi^k\right] = E\left[X^k\right] \qquad \text{(M-16)}$$

Let

$$\left.\begin{aligned} \mu(\xi) &= \mu(X) = \mu \\ \sigma^2(\xi) &= \sigma^2(X) = \sigma^2 \\ \mu_3(\xi) &= \mu_3(X) = \mu_3 \\ \mu_4(\xi) &= \mu_3(X) = \mu_4 \\ \gamma(\xi) &= \gamma(X) = \left(\mu_3 / \sigma^{3/2}\right) = \gamma \\ \lambda(\xi) &= \lambda(X) = \left(\mu_4 / \sigma^4\right) = \lambda \end{aligned}\right\} \qquad \text{(M-17)}$$

It follows that

$$\left.\begin{aligned}
&\mu(\zeta) = \mu \\
&\sigma^2(\zeta) = \left[\beta^2 + \left(1-\beta^2\right)\right]\sigma^2 = K^2\sigma^2 \\
&\mu_3(\zeta) = \left[\beta^3 + \left(1-\beta^3\right)\right]\mu_3 \\
&\mu_4(\zeta) = \left[\beta^4 + (1-\beta)^4\right]\mu_4 + 6\beta^2\left(1-\beta^2\right)\sigma^4 \\
&\gamma(\zeta) = \left\{\frac{\left[\beta^3 + \left(1-\beta^3\right)\right]}{\left[\beta^2 + \left(1-\beta^2\right)\right]^{3/2}}\right\}\gamma \\
&\lambda(\zeta) = \left\{\frac{\left[\beta^4 + (1-\beta)^4\right]\lambda + 6\beta^2\left(1-\beta^2\right)}{\left[\beta^2 + \left(1-\beta^2\right)\right]^2}\right\}
\end{aligned}\right\} \tag{M-18}$$

where

$$\sigma^2(\zeta) = \begin{cases} \sigma^2;\ if\ \beta = 1 \\ \sigma^2;\ if\ \beta = 0 \\ 5\sigma^2;\ if\ \beta = -1 \end{cases} \tag{M-19}$$

$$\gamma(\zeta) = \begin{cases} \gamma;\ if\ \beta = 1 \\ \gamma;\ if\ \beta = 0 \\ \left(8/5^{3/2}\right)\gamma;\ if\ \beta = -1 \end{cases} \tag{M-20}$$

$$\lambda(\zeta) = \begin{cases} \lambda;\ if\ \beta = 1 \\ \lambda;\ if\ \beta = 0 \\ (17\lambda + 24)/25;\ if\ \beta = -1 \end{cases} \tag{M-21}$$

If $\lambda = 3$, then $\lambda(\zeta) = 3\ \ \forall\beta$.

The solution of

$$\begin{aligned} d\,\sigma^2(\zeta)/d\beta &= (4\beta - 2)\sigma^2 \\ &= 0 \end{aligned} \tag{M-22}$$

yields $\beta = 0.5$, for which $\sigma^2_{min}(\zeta)$, the minimum value of $\sigma^2(\zeta)$, is

$$\sigma^2_{min}(\zeta) = \sigma^2/2 \tag{M-23}$$

The solution of

$$\begin{aligned} d\gamma(\zeta)/d\beta &= (2\beta-1)\gamma \\ &= 0 \end{aligned} \qquad \text{(M-24)}$$

yields $\beta = 0.5$, for which $\gamma_{min}(\zeta)$, the minimum value of $\gamma(\zeta)$, is

$$\gamma_{min}(\zeta) = \gamma/2 \qquad \text{(M-25)}$$

There are three solutions of

$$\begin{aligned} d\lambda(\zeta)/d\beta &= 4\beta(1-2\beta)(1-\beta)[\lambda-3] \\ &= 0 \end{aligned} \qquad \text{(M-26)}$$

namely, $\beta = 0$, $\beta = 0.5$ and $\beta = 1$ conditioned on $\lambda \neq 3$. The solutions $\beta = 0$ and $\beta = 1$ yield equal maximum valuea of $\lambda(\zeta)$, namely $\lambda_{max}(\zeta) = \lambda$. The solution $\beta = 0.5$ yields

$$\lambda_{min}(\zeta) = (\lambda+3)/2 \qquad \text{(M-27)}$$

See Figures M-1, M-2 and M-3.

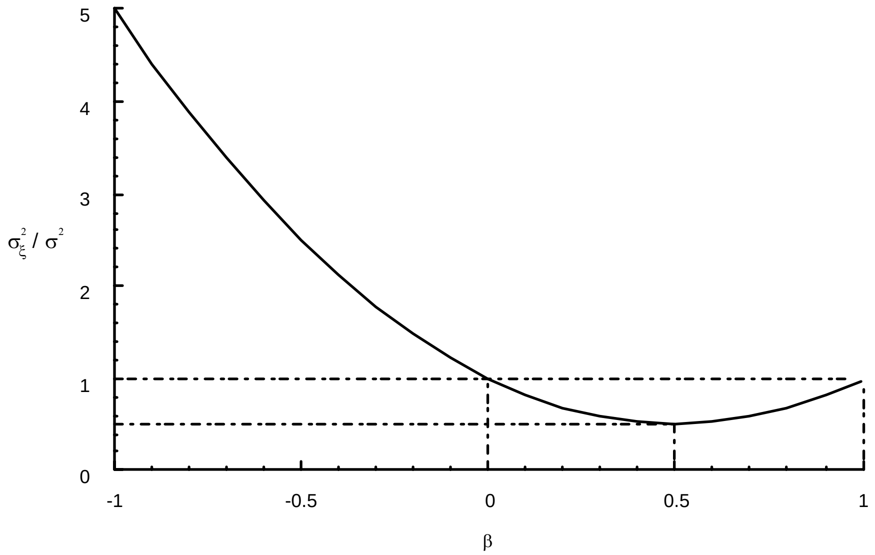


Figure M-1: Ratio of the Variance of ζ to the Variance of $\xi \equiv X$ in relation to β

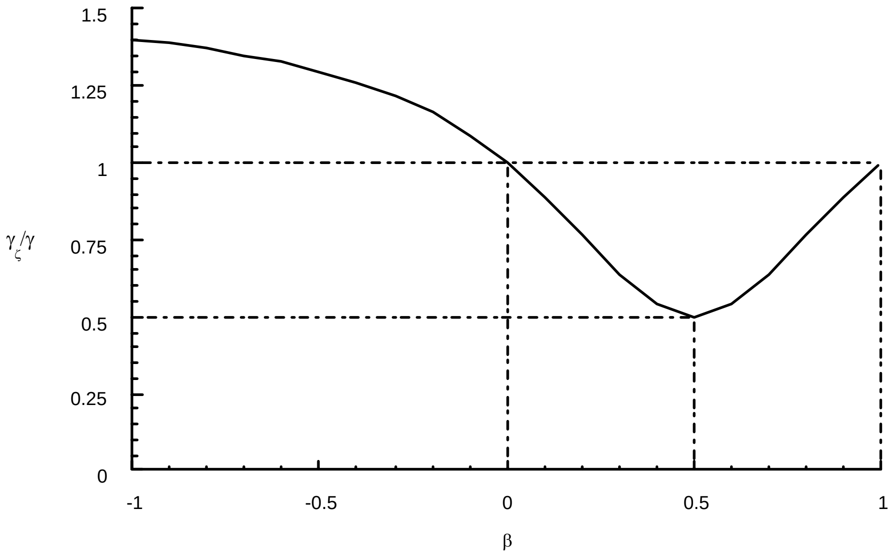


Figure M-2: Ratio of the Skewness of ζ to the Skewness of $\xi \equiv X$ in Relation to β

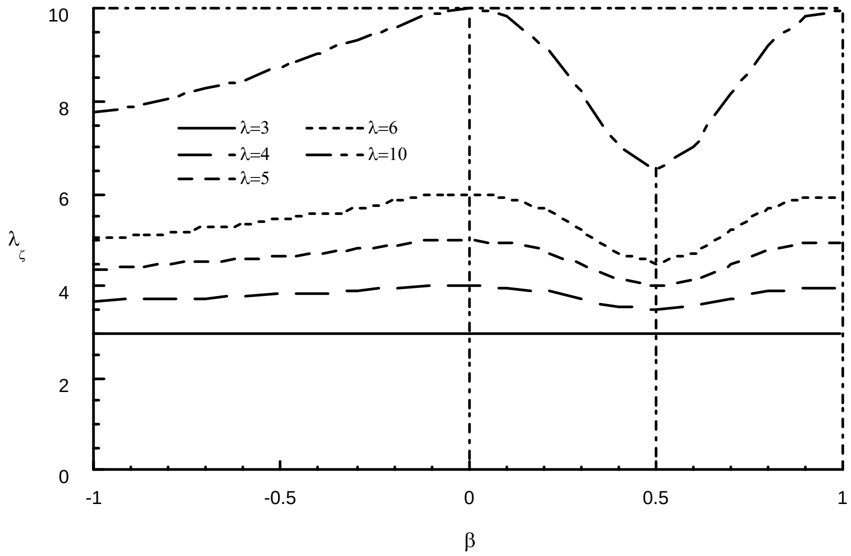


Figure M-3: Kurtosis of ζ in Relation to β Conditioned on the Kurtosis of $\xi \equiv X$

Given that X and Y are independent, the covariance of X and Y

$$\begin{aligned} Cov(X,Y) &= E[XY] - E[X]E[Y] \\ &= E[XY] - \mu^2 \\ &= 0 \end{aligned} \tag{M-28}$$

whereby, the correlation, measure of linear dependence, between X and Y is

$$\begin{aligned} \rho(X,Y) &= Cov(X,Y)/\sigma(X)\sigma(Y) \\ &= Cov(X,Y)/\sigma^2 \\ &= 0 \end{aligned} \tag{M-29}$$

The covariance between ξ and ζ is given as

$$\begin{aligned} Cov(\xi,\zeta) &= E[\xi\zeta] - \mu(\xi)\mu(\zeta) \\ &= E\left[\beta X^2 + (1-\beta)XY\right] - \mu^2 \\ &= \beta E\left[X^2 - \mu^2\right] \\ &= \beta\sigma^2 \end{aligned} \tag{M-30}$$

whereby, the correlation between ξ and ζ is given as

$$\begin{aligned}\rho(\xi,\zeta) &\equiv \rho \\ &= Cov(\xi,\zeta)/\left(\sigma(\xi)\sigma(\zeta)\right) \\ &= \beta\sigma^2 \Big/ \left\{\left[\beta^2 + (1-\beta)^2\right]^{1/2}\sigma^2\right\} \\ &= \beta \Big/ \left[\beta^2 + (1-\beta)^2\right]^{1/2}\end{aligned} \qquad \text{(M-31)}$$

where

$$\rho = \begin{cases} 1;\ if\ \beta = 1 \\ 0;\ if\ \beta = 0 \\ -1/\sqrt{5};\ if\ \beta = -1 \end{cases} \qquad \text{(M-32)}$$

Refer to Eqs. (M-17) and (M-18).

See Figure M-4.

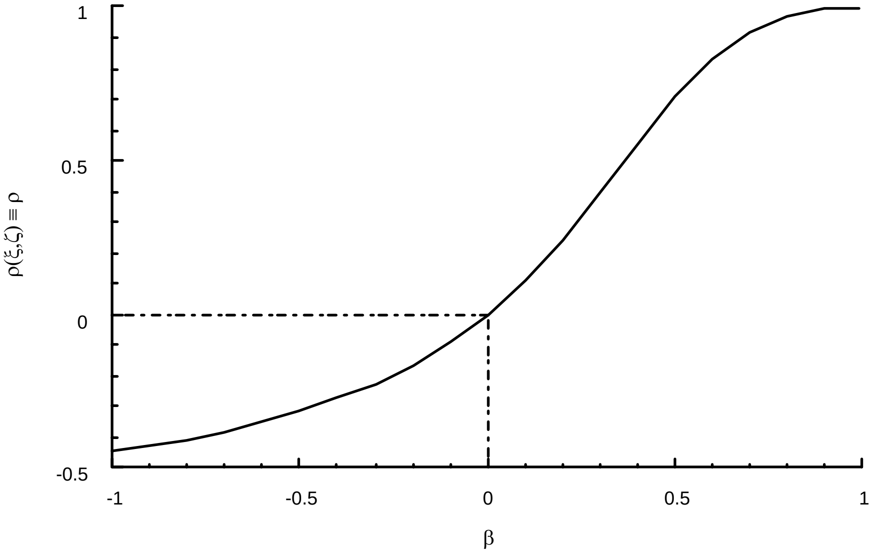


Figure M-4: Relation between Measure of Dependence, β, and Coefficient of Correlation, ρ(ξ,ζ) ≡ ρ

Normal and Log Normal Marginals: Unspecified Bivariate Form

Let X and Y be independent and identically distributed Normal random variables. The sum of normally distributed random variables is normally distributed. It follows that ξ and ζ, defined by Eqs. (M-1) and (M-2), are dependently distributed Normal random variables. See e.g. Johnson and Kotz (1970). The probability density function, $f(x)$, of X may be expressed as

$$f(x) = \left[\sqrt{2\pi\sigma^2}\right]^{-1} exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^2\right] \tag{M-33}$$

where $-\infty \le x \le \infty$.

It follows that

$$f(\xi) = f(x) = f(y) \tag{M-34}$$

where X , Y and ξ are each distributed on the interval $(-\infty, \infty)$.

The Normal distribution has the characteristics

$$\left.\begin{aligned} \mu(\xi) &= \mu(X) = \mu \\ \sigma^2(\xi) &= \sigma^2(X) = \sigma^2 \\ \mu_3(\xi) &= \mu_3(X) = 0 \\ \mu_4(\xi) &= \mu_4(X) = 3\sigma^4 \\ \gamma(\xi) &= \gamma(X) = 0 \\ \lambda(\xi) &= \lambda(X) = 3 \end{aligned}\right\} \tag{M-35}$$

It follows that

$$\left.\begin{aligned}
&\mu(\zeta)=\mu\\
&\sigma^2(\zeta)=\left[\beta^2+(1-\beta)^2\right]\sigma^2\\
&\mu_3(\zeta)=0\\
&\mu_4(\zeta)=3\left[\beta^2+(1-\beta)^2\right]^2\sigma^4\\
&\gamma(\zeta)=0\\
&\lambda(\zeta)=3
\end{aligned}\right\} \tag{M-36}$$

where ζ is distributed on the interval $(-\infty,\infty)$.

The correlation between ξ and ζ is denoted as $\rho(\xi,\zeta)=\rho_N$, where the subscript N identifies the marginal distributions, i.e. the distributions of ξ and ζ as being Normal distributes. The correlation $\rho(\xi,\zeta)$ is given by Eq. (M-31).

Define

$$\phi=exp(\xi) \tag{M-37}$$

$$\varphi=exp(\zeta) \tag{M-38}$$

In the following discussions, ξ and ζ denote random variables in log space, whereby ϕ and φ denote random variables in real space.

The relation between ξ and ζ is given by

$$\zeta=\beta\xi+(1-\beta)Y \tag{M-39}$$

whereby,

$$\begin{aligned}
\varphi&=exp(\zeta)\\
&=exp\left[\beta X+(1-\beta Y)\right]\\
&=exp(\beta X)exp\left[(1-\beta)Y\right]\\
&=exp(\beta\xi)exp\left[(1-\beta)Y\right]
\end{aligned} \tag{M-40}$$

Define

$$
\begin{aligned}
E\left[\phi^{\nu}\right] &= E\left[exp(\nu\xi)\right] \\
&= E\left[exp(\nu X)\right] \\
&= \frac{1}{\sqrt{2\pi}\,\sigma}\int_{-\infty}^{\infty} exp(\nu x)\, exp\left[-\frac{1}{2}\left(\frac{x-\mu}{\sigma}\right)^{2}\right]dx \\
&= \frac{exp\left(\nu\mu+\nu^{2}\sigma^{2}/2\right)}{\sqrt{2\pi}}\int_{-\infty}^{\infty} exp\left[-\frac{1}{2}(z-\nu\sigma)^{2}\right]dz \\
&= exp\left(\nu\mu+\nu^{2}\sigma^{2}/2\right)
\end{aligned}
\tag{M-41}
$$

where $z=(x-\mu)/\sigma$.

It follows that

$$
\left.
\begin{aligned}
\mu(\phi) &= exp\left(\mu+\sigma^{2}/2\right) \\
\sigma^{2}(\phi) &= \left\{exp\left(2\mu+\sigma^{2}\right)\right\}\left\{exp\left(\sigma^{2}\right)-1\right\} \\
\mu_{3}(\phi) &= \left\{exp\left(3\mu+3\sigma^{2}/2\right)\right\}\left\{exp\left(3\sigma^{2}\right)-3\,exp\left(\sigma^{2}\right)+2\right\} \\
&= \left\{exp\left(3\mu+3\sigma^{2}/2\right)\right\}\left\{\eta^{6}+3\eta^{4}\right\} \\
\mu_{4}(\phi) &= \left\{exp\left(4\mu+2\sigma^{2}\right)\right\}\left\{exp\left(6\sigma^{2}\right)-4\,exp\left(3\sigma^{2}\right)+6\,exp\left(\sigma^{2}\right)-3\right\} \\
&= \left\{exp\left(4\mu+2\sigma^{2}\right)\right\}\left\{\eta^{12}+6\eta^{10}+15\eta^{8}+16\eta^{6}+3\eta^{4}\right\} \\
\gamma(\phi) &= \eta^{3}+3\eta \\
\lambda(\phi) &= \eta^{8}+6\eta^{6}+15\eta^{4}+16\eta^{2}+3
\end{aligned}
\right\}
\tag{M-42}
$$

where

$$
\eta^{2}=exp\left(\sigma^{2}\right)-1 \tag{M-43}
$$

Define

$$
\begin{aligned}
E\left[\varphi^{\nu}\right] &= E\left[exp\left(\nu\zeta\right)\right] \\
&= E\left[exp\left(\nu\beta X + \nu(1-\beta)Y\right)\right] \\
&= E\left[exp\left(\nu\beta X\right)\right]E\left[\nu(1-\beta)Y\right] \\
&= \left\{\int_{-\infty}^{\infty} exp\left(\nu\beta x\right)f(x)dx\right\}\left\{\int_{-\infty}^{\infty} exp\left(\nu(1-\beta)\right)f(y)dy\right\} \\
&= exp\left(\nu\beta\mu + (\nu\beta\sigma)^2/2\right)exp\left(\nu(1-\beta)\mu + \left(\nu(1-\beta)\sigma\right)^2/2\right) \\
&= exp\left(\nu\mu + \nu^2\left(\beta^2 + (1-\beta)^2\right)\sigma^2/2\right) \\
&= exp\left(\nu\mu + (\nu K\sigma)^2/2\right)
\end{aligned}
\tag{M-44}
$$

It follows that

$$
\left.
\begin{aligned}
\mu(\varphi) &= exp\left(\mu + (K\sigma)^2/2\right) \\
\sigma^2(\varphi) &= \left\{exp\left(2\mu + K^2\sigma^2\right)\right\}\left\{exp\left(K^2\sigma^2\right) - 1\right\} \\
\mu_3(\varphi) &= \left\{exp\left(3\mu + 3K^2\sigma^2/2\right)\right\}\left\{exp\left(3K^2\sigma^2\right) - 3\,exp\left(K^{2\sigma^2}\right) + 2\right\} \\
&= \left\{exp\left(3\mu + 3K^2\sigma^2/2\right)\right\}\left\{\omega^6 + 3\omega^4\right\} \\
\mu_4(\varphi) &= \left\{exp\left(4\mu + 2K^2\sigma^2\right)\right\}\left\{\begin{matrix} exp\left(6K^2\sigma^2\right) - 4\,exp\left(4K^2\sigma^2\right) \\ +6\,exp\left(K^2\sigma^2\right) - 3 \end{matrix}\right\} \\
&= \left\{exp\left(4\mu + 2K^2\sigma^2\right)\right\}\left\{\omega^{12} + 6\omega^{10} + 15\omega^8 + 16\omega^6 + 3\omega^4\right\} \\
\gamma(\varphi) &= \omega^3 + 3\omega \\
\lambda(\varphi) &= \omega^8 + 6\omega^6 + 15\omega^4 + 16\omega^2 + 3
\end{aligned}
\right\}
\tag{M-45}
$$

where

$$
\omega^2 = exp\left(K^2\sigma^2\right) - 1 \tag{M-46}
$$

The covariance between ϕ and φ is given by

$$\begin{aligned} Cov(\phi, \varphi) &= E[\phi\varphi] - E[\phi]E[\varphi] \\ &= E[exp(\xi + \zeta)] - E[\phi]E[\varphi] \\ &= \{exp(2\mu + (1 + K^2)\sigma/2)\}\{exp(\beta\sigma^2) - 1\} \end{aligned} \tag{M-47}$$

whereby, the correlation between ϕ and φ may be expressed as

$$\begin{aligned} \rho(\phi, \varphi) &\equiv \rho_{LN} \\ &= Cov(\phi, \varphi)/\sigma(\phi)\sigma(\varphi) \\ &= \frac{\{exp(\beta\sigma^2) - 1\}}{\{exp(\sigma^2) - 1\}^{1/2}\{exp(K^2\sigma^2) - 1\}^{1/2}} \\ &= \frac{\{exp(\rho_N K\sigma^2) - 1\}}{\{exp(\sigma^2) - 1\}^{1/2}\{exp(K^2\sigma^2) - 1\}^{1/2}} \end{aligned} \tag{M-48}$$

where

$$\rho_{LN} = \begin{cases} 1;\ if\ \beta = 1 \\ 0;\ if\ \beta = 0 \\ -[exp(\sigma^2) - 1]^{1/2} \Big/ \{exp(\sigma^2)[exp(5\sigma^2) - 1]^{1/2}\}\ if\ \beta < 0 \end{cases} \tag{M-49}$$

Refer to Eq. (M-31).

The correlation in real space, ρ_{LN}, is structurally a function of three parameters in log space, namely the correlation $\rho_N \equiv \rho$, the scale parameter of ξ, σ, and the scale parameter of ζ, $K\sigma$. Refer to Eqs. (M-17) and (M-18). However, ρ_{LN} has only two degrees of freedom, namely the scale parameter σ and the weighting factor β. The factor β defines ρ_N. Given σ, β defines the scale parameter of ζ, namely $K\sigma$.

The scale parameters σ and $K\sigma$ determine the shape parameters, i.e. the skewness and kurtosis, of the distributions of ξ and ζ. If $\gamma(\phi) = \gamma(\varphi)$, then $K = 1$, implying that $\beta = 0$ or $\beta = 1$. If $\beta = 0$, then $\rho_N = 0$ and consequently $\rho_{LN} = 0$. If $\beta = 1$, then $\rho_N = 1$ and consequently $\rho_{LN} = 1$. Unless $\gamma(\phi) = \gamma(\varphi)$, ϕ and φ cannot be perfectly correlated.

As $\sigma^2 \to 0$, the correlation in real space, ρ_{LN}, approaches the correlation in log space, ρ_N, given by Eq. (M-31) where $\rho \equiv \rho_N$. As σ^2 increases, ρ_{LN} decreases (decreases) $\forall\ \beta > 0$ (<0). As $\sigma^2 \to \infty$, $\rho_{LN} \to 0\ \forall\ \beta < 1$. See Figure M-5.

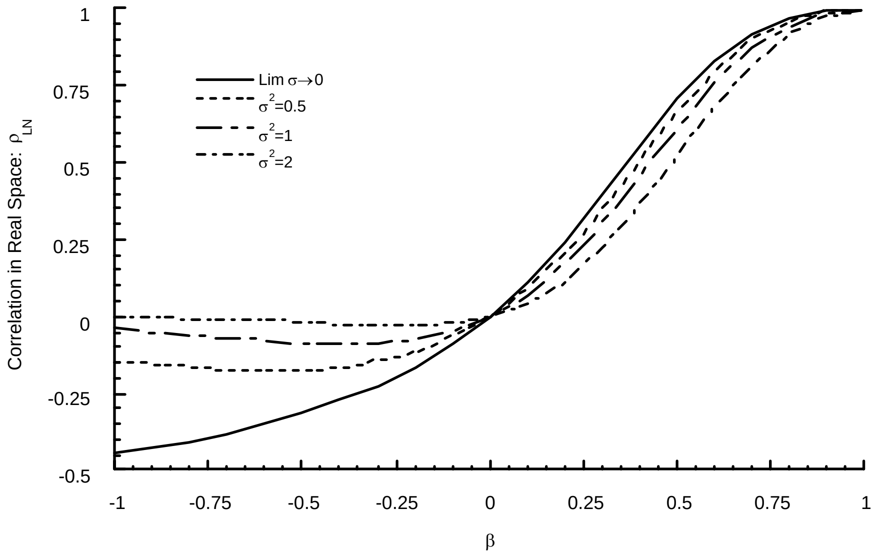


Figure M-5: Relation between ρ_{LN} and β Conditioned on σ^2 ($\rho_{LN} \to \rho_N$ as $\sigma^2 \to 0$)

The relation between the correlation in log space, ρ_N, and the correlation in real space, ρ_{LN}, is shown in Figure M-6.

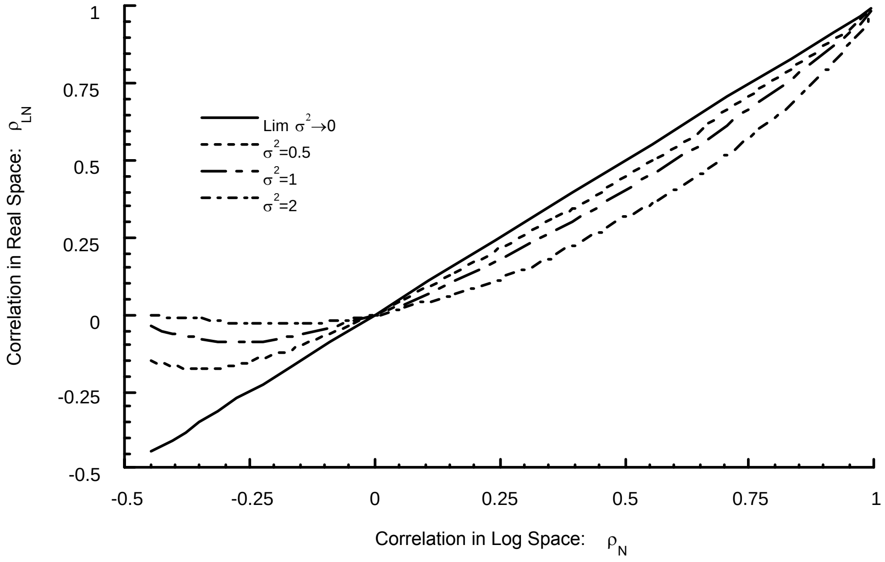


Figure M-6: Relation between Correlation in Real and Log Space with Normal Marginal Distributions in Log Space Conditioned on σ^2

From Figure 6, it is seen that in absolute value, the correlation in log space is greater than the correlation in real space. As $\sigma^2 \to \infty$, $\rho_{LN} \to 0 \;\; \forall \;\; \rho_N < 1$.

The relation between ϕ and φ may be expressed as

$$\begin{aligned}\varphi &= exp(\zeta) \\ &= exp\left[\beta\xi + (1-\beta)Y\right] \\ &= \phi^{\beta}\, exp\left[(1-\beta)Y\right]\end{aligned} \qquad \text{(M-50)}$$

Normal and Log Normal Marginals: Specified Bivariate Normal

In log space, let the random variables ξ and ζ be jointly distributed as bivariate Normal. The distributions of ξ and ζ , i.e. the marginal distributions, are Normal. The relation between ξ and ζ may be linearly expressed as

$$\zeta = m_2 + \tilde{\beta}(\xi - m_1) + \left(1 - \tilde{\rho}_N^2\right)^{1/2} b_2 \varepsilon \qquad \text{(M-51)}$$

where ε is normally distributed with zero mean and unit variance independently of ξ, and

$$\left.\begin{aligned} m_1 &= \mu(\xi) \\ m_2 &= \mu(\zeta) \\ b_1^2 &= \sigma^2(\xi) \\ b_2^2 &= \sigma^2(\zeta) \\ \tilde{\beta} &= \tilde{\rho}_N \sigma(\zeta) / \sigma(\xi) \end{aligned}\right\} \qquad \text{(M-52)}$$

The relation between ξ and ζ defined by Eq. (M-51) references the regression of ζ on ξ, where $\tilde{\beta}$ denotes the regression coefficient and $\tilde{\rho}_N$, the correlation between ξ and ζ.

In real space

$$\phi = exp(\xi) \qquad \text{(M-53)}$$

$$\varphi = exp(\zeta) \qquad \text{(M-54)}$$

whereby the low order moment characteristics of ϕ are

$$
\left.\begin{aligned}
\mu(\phi) &= exp\left(m_1 + b_1^2/2\right) \\
\sigma^2(\phi) &= \left\{exp\left(2m_1\right) + b_1^2\right\}\left\{exp\left(b_1^2\right) - 1\right\} \\
\mu_3(\phi) &= \left\{exp\left(3m_1\right) + 3b_1^2/2\right\}\left\{exp\left(3b_1^2\right) - 3\,exp\left(b_1^2\right) + 2\right\} \\
&= \left\{exp\left(3m_1\right) + 3b_1^2/2\right\}\left\{v^6 + 3\,v^4\right\} \\
\mu_{4\gamma}(\phi) &= \left\{exp\left(4m_1 + 2b_1^2\right)\right\}\left\{exp\left(6b_1^2\right) - 4\,exp\left(3b_1^2\right) + 6\,exp\left(b_1^2\right) - 3\right\} \\
&= \left\{exp\left(4m_1 + 2b_1^2\right)\right\}\left\{v^{12} + 6\,v^{10} + 15\,v^8 + 16\,v^6 + 3\,v^4\right\} \\
\gamma(\phi) &= v^3 + 3\,v \\
\lambda(\phi) &= v^8 + 6\,v^6 + 15\,v^4 + 16\,v^2
\end{aligned}\right\} \qquad \text{(M-55)}
$$

where

$$v^2 = exp\left(b_1^2\right) - 1 \qquad \text{(M-56)}$$

The low order moment characteristics of ζ are given by Eq. (M-55) with ϕ replaced by φ, 1 replaced by 2 and v^2, defined by Eq. (M-56), replaced by

$$\tau^2 = exp\left(b_2\right) - 1 \qquad \text{(M-57)}$$

The covariance between ϕ and φ is defined as

$$
\begin{aligned}
Cov(\phi, \varphi) &= E[\phi\varphi] - E[\phi]E[\varphi] \\
&= \left\{exp\left[\left(m_1 + m_2\right) + \left(b_1^2 + b_2^2\right)/2\right]\right\}\left\{exp\left(\tilde{\rho}_N b_1 b_2\right) - 1\right\}
\end{aligned} \qquad \text{(M-58)}
$$

whereby the correlation between ϕ and φ, i.e. the correlation in real space, is

$$
\begin{aligned}
\rho(\phi, \varphi) &= \tilde{\rho}_{LN} \\
&= \frac{exp\left(\tilde{\rho}_N b_1 b_2\right) - 1}{\left\{exp\left(b_1^2\right) - 1\right\}^{1/2}\left\{exp\left(b_2^2\right) - 1\right\}^{1/2}}
\end{aligned} \qquad \text{(M-59)}
$$

For $b_1 \neq b_2$

$$\tilde{\rho}_{LN} = \begin{cases} \dfrac{exp\left(b_1 b_2\right) - 1}{\left\{exp\left(b_1^2\right) - 1\right\}^{1/2} \left\{exp\left(b_2^2\right) - 1\right\}^{1/2}} ; if\ \tilde{\rho}_N = 1 \\ 0;\ if\ \tilde{\rho}_N = 0 \\ \dfrac{1 - exp\left(b_1 b_2\right)}{\left\{exp\left(b_1 b_2\right)\right\}\left\{exp\left(b_1^2\right) - 1\right\}^{1/2} \left\{exp\left(b_2^2\right) - 1\right\}^{1/2}} ; if\ \tilde{\rho}_N = -1 \end{cases} \tag{M-60}$$

For $b_1 = b_2$

$$\tilde{\rho}_{LN} = \begin{cases} 1;\ if\ \tilde{\rho}_N = 1 \\ 0;\ if\ \tilde{\rho}_N = 0 \\ exp(-1) if\ \tilde{\rho}_N = -1 \end{cases} \tag{M-61}$$

The correlation in real space, $\tilde{\rho}_{LN}$, is structurally defined by three parameters in log space, namely the correlation $\tilde{\rho}_N$ and the scale parameters b_1 and b_2 of the distributions of ξ and ζ. The scale parameters b_1 and b_2 have no bearing on $\tilde{\rho}_N$.

In general, $\tilde{\rho}_{LN} \leq \tilde{\rho}_N$ for $\tilde{\rho}_N \geq 0$, where equality attains for either $\tilde{\rho}_N = 0$ or $\tilde{\rho}_N = 1$, and $\tilde{\rho}_{LN} > \tilde{\rho}_N$ for $\tilde{\rho}_N < 0$. If $b_1 = b_2 = b$, then as b increases, whereby $\gamma(\phi) = \gamma(\varphi)$ increases, the correlation in real space, $\tilde{\rho}_{LN}$, decreases relative to a given value of $\tilde{\rho}_N$. See Figure M-7.

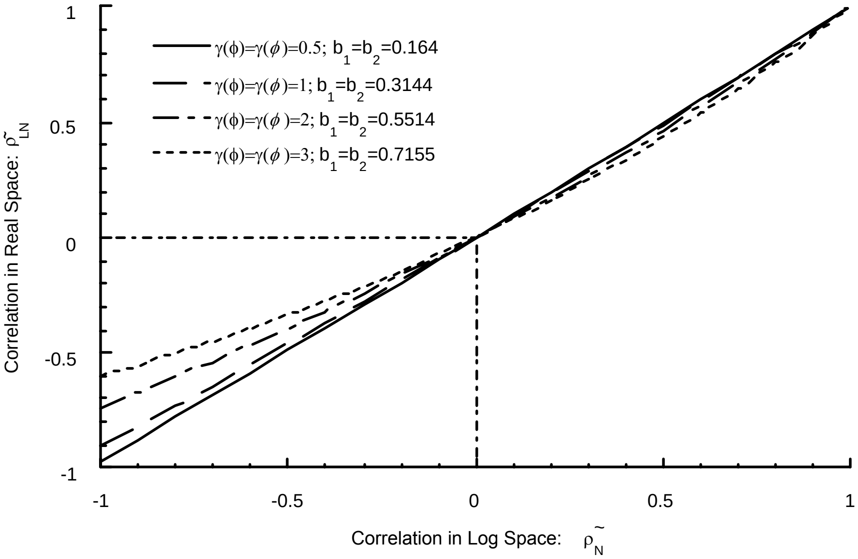


Figure M-7: Relation between Correlations in Real and Log Space Conditioned on Skewness in Real Space Equivalent to Scale in Log Space

If $b_1 \neq b_2$, in which case $\gamma(\phi) \neq \gamma(\varphi)$, then $\tilde{\rho}_{LN}$ decreases relative to a given value of $\tilde{\rho}_N$ as $|b_1 - b_2|$ increases, or equivalently as $|\gamma(\phi) - \gamma(\varphi)|$. See Figure M-8.

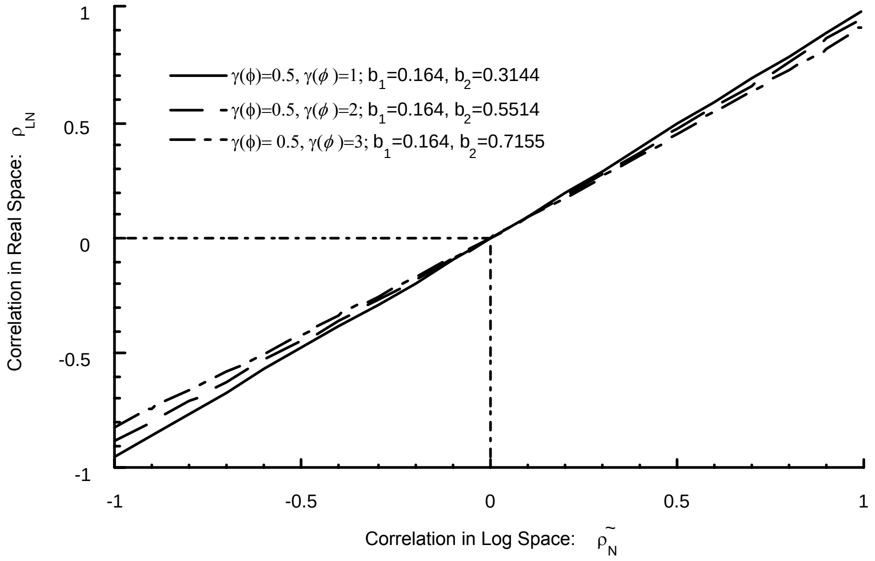


Figure M-8: Relation between Correlations in Real and Log Space given Unequal Skews in Real Space, Equivalently Unequal Scales in Log Space

Let $b_1 = b$ and $b_2 = Lb$, where $L > 0$. Eq.(M-58) may be expressed as

$$\tilde{\rho}_{LN} = \frac{exp\left(\tilde{\rho}_N L b^2\right) - 1}{\left\{exp\left(b^2\right) - 1\right\}^{1/2} \left\{exp\left(L^2 b^2\right) - 1\right\}^{1/2}} \tag{M-62}$$

which is parallel in structure to the relation between the real and log space correlation based on the weighted linear combination procedure given by Eq. (M-48). As noted above, $\tilde{\rho}_{LN}$ is determined with three degrees of freedom, whereas ρ_{LN} is determined with two degrees of freedom. Let $b = \sigma$ and $L = K = 1$. Then β equals either 0 or 1, so that ρ_N equals 0 and 1, whereby ρ_{LN} equals 0 or 1. However, $\tilde{\rho}_{LN}$ may assume any value within the interval $\left\{\left(exp(-b) - 1\right) / \left(exp\left(b^2\right) - 1\right)\left(exp(b) - 1\right) / exp\left(b^2\right) - 1\right\}$. Let $b = \sigma$ and $L = K$. Then for $\rho_N = \tilde{\rho}_N$, $\rho_{LN} = \tilde{\rho}_{LN}$.

Pearson Type III and Log Pearson Type III Marginals: Unspecified Bivariate Form

In the following discussions, the Pearson Type III and Log Pearson Type III distributions are referred to as the Pearson and Log Pearson distributions. In log space, let X and Y be independent and identically distributed Pearson random variables. The sum of Pearson distributed random variables is distributed as Pearson. Thus, ξ and ζ, defined by Eqs. (M-1) and (M-2) are dependently distributed Pearson random variables in log space. The probability density function, $f(x)$, of X may be expressed as

$$f(x)=\frac{1}{|a|\Gamma(b)}\left(\frac{(x-m)}{a}\right)^{b-1} exp\left[-\left(\frac{(x-m)}{a}\right)\right] \tag{M-63}$$

where $m \le x \le \infty$ if $a > 0$ or $-\infty < x \le m$ if $a < 0$. If $a > 0$, then $f(x)$ is characterized by positive skewness, and if $a < 0$, then $f(x)$ is characterized by negative skewness.

The Pearson distribution has the characteristics

$$\left.\begin{aligned}
\mu(\xi) &= m + ab \\
\sigma^2(\xi) &= ab^2 \\
\gamma(\xi) &= \frac{2a}{|a|\sqrt{b}} \\
\lambda(\xi) &= 3 + 6/b
\end{aligned}\right\} \tag{M-64}$$

It follows that

$$\left.\begin{aligned}
\mu(\zeta) &= m + ab \\
\sigma^2(\zeta) &= \left[\beta^2 + (1-\beta)^2\right]ab^2 \\
\gamma(\zeta) &= \left\{\frac{\left[\beta^3 + (1-\beta)^3\right]}{\left[\beta^2 + (1-\beta)^2\right]^{3/2}}\right\}\left\{\frac{2a}{|a|\sqrt{b}}\right\} \\
\lambda(\zeta) &= \left\{\frac{\left[\beta^4 + \left(1-\beta^4\right)\right]\left[3 + 6/b\right] + 6\beta^2\left(1-\beta^2\right)}{\left[\beta^2 + (1-\beta)^2\right]^2}\right\}
\end{aligned}\right\} \tag{M-65}$$

The correlation between ξ and ζ is denoted as $\rho(\xi,\zeta) = \rho_P$, where the subscript P identifies the Pearson distribution, i.e. the distributions of ξ and ζ as being Pearson distributions. The correlation $\rho(\xi,\zeta)$ is given by Eq. (M-31).

Define

$$\begin{aligned} E\left[\phi^{v}\right] &= E\left[exp(v\xi)\right] \\ &= E\left[exp(vX)\right] \\ &= \int_{m}^{\infty} exp(vx) f(x) dx \\ &= exp(vm)\left[(1-va)^{-b}\right] \end{aligned} \qquad \text{(M-66)}$$

where $(1-va) > 0$ and $f(x)$ is given by Eq. (M-63).

It follows that

$$\left.\begin{aligned} \mu(\phi) &= exp(m)\left[(1-a)^{-b}\right] \\ \sigma^{2}(\phi) &= exp(2m)\left[(1-2a)^{-b} - (1-a)^{-2b}\right] \\ \gamma(\phi) &= \frac{\left[(1-3a)^{-b} - 3(1-a)^{-b}(1-2a)^{-b} + 2(1-a)^{-3b}\right]}{\left[(1-2a)^{-b} - (1-a)^{-2b}\right]^{3/2}} \\ \lambda(\phi) &= \frac{\begin{bmatrix} (1-4a)^{-b} - 4(1-a)^{-b}(1-3a)^{-b} + 6(1-a)^{-2b}(1-2a)^{-b} \\ -3(1-a)^{-4b} \end{bmatrix}}{\left[(1-2a)^{-b} - (1-a)^{-2b}\right]^{2}} \end{aligned}\right\} \qquad \text{(M-67)}$$

where

$$\left.\begin{array}{l} \mu(\phi) \\ \sigma^{2}(\phi) \\ \gamma(\phi) \\ \lambda(\phi) \end{array}\right\} is\ defined \left\{\begin{array}{l} if\ a < 1 \\ if\ a < 1/2 \\ if\ a < 1/3 \\ if\ a < 1/4 \end{array}\right.$$

Define

$$
\begin{aligned}
E\left[\varphi^{v}\right] &= E\left[exp\left(v\zeta\right)\right] \\
&= E\left[exp\left(v\beta X + v\left(1-\beta\right)Y\right)\right] \\
&= E\left[exp\left(v\beta X\right)\right]E\left[exp\left(v\left(1-\beta\right)Y\right)\right] \\
&= \int_{m}^{\infty} exp\left(v\beta x\right)f\left(x\right)dx \int_{m}^{\infty} exp\left(v\left(1-\beta\right)y\right)f\left(y\right)dy \qquad \text{(M-68)} \\
&= \left\{exp\left(v\beta m\right)\left[\left(1-v\beta a\right)^{-b}\right]\right\}\left\{exp\left(v\left(1-\beta\right)m\right)\left[\left(1-v\left(1-\beta\right)a\right)^{-b}\right]\right\} \\
&= exp\left(vm\right)\left[\left(1-v\beta a\right)^{-b}\right]\left[\left(1-v\left(1-\beta\right)a\right)^{-b}\right]
\end{aligned}
$$

It follows that

$$
\left.
\begin{aligned}
\mu(\varphi) &= exp(m)\left[\left((1-\beta a)(1-(1-\beta)a)\right)^{-b}\right] \\
\sigma^2(\varphi) &= exp(2m)\left[\begin{array}{l}\left((1-2\beta a)(1-2(1-\beta)a)\right)^{-b} \\ -\left((1-\beta a)(1-(1-\beta)a)\right)^{-2b}\end{array}\right] \\
\gamma(\varphi) &= \frac{\left[\begin{array}{l}\left((1-3\beta a)(1-3(1-\beta)a)\right)^{-b} \\ -3\left[\left((1-\beta a)(1-(1-\beta)a)\right)^{-b}\right]\left[\left((1-2\beta a)(1-2(1-\beta)a)\right)^{-b}\right] \\ +2\left[\left((1-\beta a)(1-(1-\beta)a)\right)^{-3b}\right]\end{array}\right]}{\left[\begin{array}{l}\left((1-2\beta a)(1-2(1-\beta)a)\right)^{-b} \\ -\left((1-\beta a)(1-(1-\beta)a)\right)^{-2b}\end{array}\right]^{3/2}} \\
\lambda(\varphi) &= \frac{\left\{\begin{array}{l}\left((1-4\beta a)(1-4(1-\beta)a)\right)^{-b} \\ -4\left[\left((1-\beta a)(1-(1-\beta)a)\right)^{-b}\right]\left[\left((1-3\beta a)(1-3(1-\beta)a)\right)^{-b}\right] \\ +6\left[\left((1-\beta a)(1-(1-\beta)a)\right)^{-2b}\right]\left[\left((1-2\beta a)(1-2(1-\beta)a)\right)^{-b}\right] \\ -3\left[\left((1-\beta a)(1-(1-\beta)a)\right)^{-4b}\right]\end{array}\right\}}{\left[\begin{array}{l}\left((1-2\beta a)(1-2(1-\beta)a)\right)^{-b} \\ -\left((1-\beta a)(1-(1-\beta)a)\right)^{-2b}\end{array}\right]^{2}}
\end{aligned}
\right\}
\qquad \text{(M-69)}
$$

where

$$
\left.\begin{array}{l}\mu(\varphi) \\ \sigma^2(\varphi) \\ \gamma(\varphi) \\ \lambda(\varphi)\end{array}\right\} is\ defined \left\{\begin{array}{l} if\ \beta a < 1 \\ if\ \beta a < 1/2 \\ if\ \beta a < 1/3 \\ if\ \beta a < 1/4 \end{array}\right.
$$

The covariance of ϕ and φ is

$$\begin{aligned} Cov(\phi,\varphi) &= E[\phi\varphi]-E[\phi]E[\varphi] \\ &= E\Big[exp\big(X+\beta X+(1-\beta)Y\big)\Big]-E[exp(X)]E\Big[exp\big(\beta X+(1-\beta)Y\big)\Big] \\ &= E\Big[exp\big((1+\beta)X\big)\Big]E\Big[exp\big((1-\beta)Y\big)\Big] \\ &\quad -E[exp(X)]E\Big[exp\big(\beta X+(1-\beta)Y\big)\Big] \\ &= exp(2m)\left\{\begin{array}{l}[1-a(1+\beta)]^{-b}[1-a(1-\beta)]^{-b} \\ -(1-a)^{-b}(1-a\beta)^{-b}[1-a(1-\beta)]^{-b}\end{array}\right\} \end{aligned} \tag{M-70}$$

and therefore,the correlation in real space, $\rho_{LP}(\phi,\varphi)\equiv\rho_{LP}$ is given by

$$\begin{aligned} \rho_{LP} &= \frac{Cov(\phi,\varphi)}{\sigma(\phi)\sigma(\varphi)} \\ &= \frac{\left\{[1-a(1-\beta)]^{-b}\right\}\left\{[1-a(1+\beta)]^{-b}-(1-a)^{-b}(1-a\beta)^{-b}\right\}}{\left\{(1-2a)^{-b}-(1-a)^{-2b}\right\}^{1/2}\left\{\begin{array}{l}(1-2a\beta)^{-b}\big(1-2a(1-\beta)\big)^{-b} \\ -(1-a\beta)^{-2b}[1-a(1-\beta)]^{-2b}\end{array}\right\}^{1/2}} \end{aligned} \tag{M-71}$$

where

$$\rho_{LP}=\begin{cases} 1;\ if\ \beta=1 \\ 0;\ if\ \beta=0 \\ A/B;\ if\ \beta=-1 \end{cases} \tag{M-72}$$

where

$$\left.\begin{array}{l} A \\ B \end{array}\right\}=\begin{cases} (1-2a)^{-b}\left[1-(1-a^2)^{-b}\right] \\ \left[(1-2a)^{-b}-(1-a)^{-2b}\right]^{1/2}\left[(1-2a)^{-b}-(1-4a)^{-b}\right]^{1/2} \end{cases} \tag{M-73}$$

The correlation in real space is determined with three degrees of freedom, namely log space parameters of scale, a, the shape parameter, b, and the weighting factor, β. The weighting factor defines the log space correlation, $\rho_P \equiv \rho$, given by Eq. (M-31). The shape parameter, b, defines the log space skewness. The real space skewness is defined by the parameters a and b.

In hydrologic studies based on the Log-Pearson distribution, it is assumed that skewness exists in real space, and empirical results suggests that the skewness in log space is negative.